

Polyèdres composites

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Topologie structurale #13, 1986

Structural Topology #13, 1986

Composite Polyhedra

Dômes stéréométriques

Les polyèdres composites constituent une catégorie de solides réguliers qui n'étaient pas jusqu'à maintenant répertoriés en tant qu'êtres géométriques. Certains, étant entièrement triangulés, forment d'emblée une composition isostatique. D'autres, vallonnés ou ondulés, peuvent être utilisés comme coupôles lobées ou comme voûtes stalactites. Cette conception architectonique originale, sous le titre de *Procédé de construction de dômes stéréométriques* (Figure 1), a fait objet de brevets de propriété industrielle (français n° 1.379.636, américain n° 3.341.989, anglais n° 1.060.867 et allemand n° 1.459.933). Ces textes, datés de 1963, font état de configurations de second ou n-ième ordre, étant donné les diverses possibilités d'enrichissement des articulations par des transformations supplémentaires, notamment: repyramidation des faces nouvellement obtenues, ou surpyramidation des faces triangulaires par des quarts de tétraèdre à triangles isocèles de $109^{\circ}28'$, ou substitution de surfaces gauches (paraboloïdes hyperboliques) aux vallées tétraédriques. Déjà dans le passé de semblables études furent entreprises par Wentzel Jamnitzer, géomètre, orfèvre et constructeur d'instruments scientifiques, né à Vienne en 1508. Toutefois son ouvrage, édité à Nuremberg en 1568, n'est pas systématique et encore moins exhaustif; il contient par contre d'autres configurations régulières dérivées de règles de jeu différentes (Jamnitzer 1964).

Stereometric Domes

The composite polyhedra constitute a group of regular solids which until recently had not been catalogued as geometrical objects. Some, being completely triangulated, make up isostatic compositions. Others, with valleys or undulations, can be used as lobed cupolas or as stalactitic vaults. This original architectural conception, which appeared under the title *Procédé de Construction de Dômes Stéréométriques* (Figure 1), is an industrial patent (French no. 1.379.636; American no. 3.341.989; English no. 1.060.867 and German no. 1.459.933). These texts, dating from 1963, contain configurations of second or n-th order, given the various possibilities for enrichment of the articulations by supplementary transformations, namely: restellation of the new faces, or superstellation of the triangular faces by tetrahedral vertices composed of isosceles triangles of $109^{\circ}28'$, or substitution of warped surfaces (hyperbolic paraboloids) into tetrahedral valleys. In the past similar studies were undertaken by Wentzel Jamnitzer, a geometer, goldsmith and manufacturer of scientific instruments, born in Vienna in 1508. His work, published in Nuremberg in 1568, is not systematic and is even less exhaustive; on the other hand, it contains some other regular configurations derived by different rules of transformation (Jamnitzer 1964).

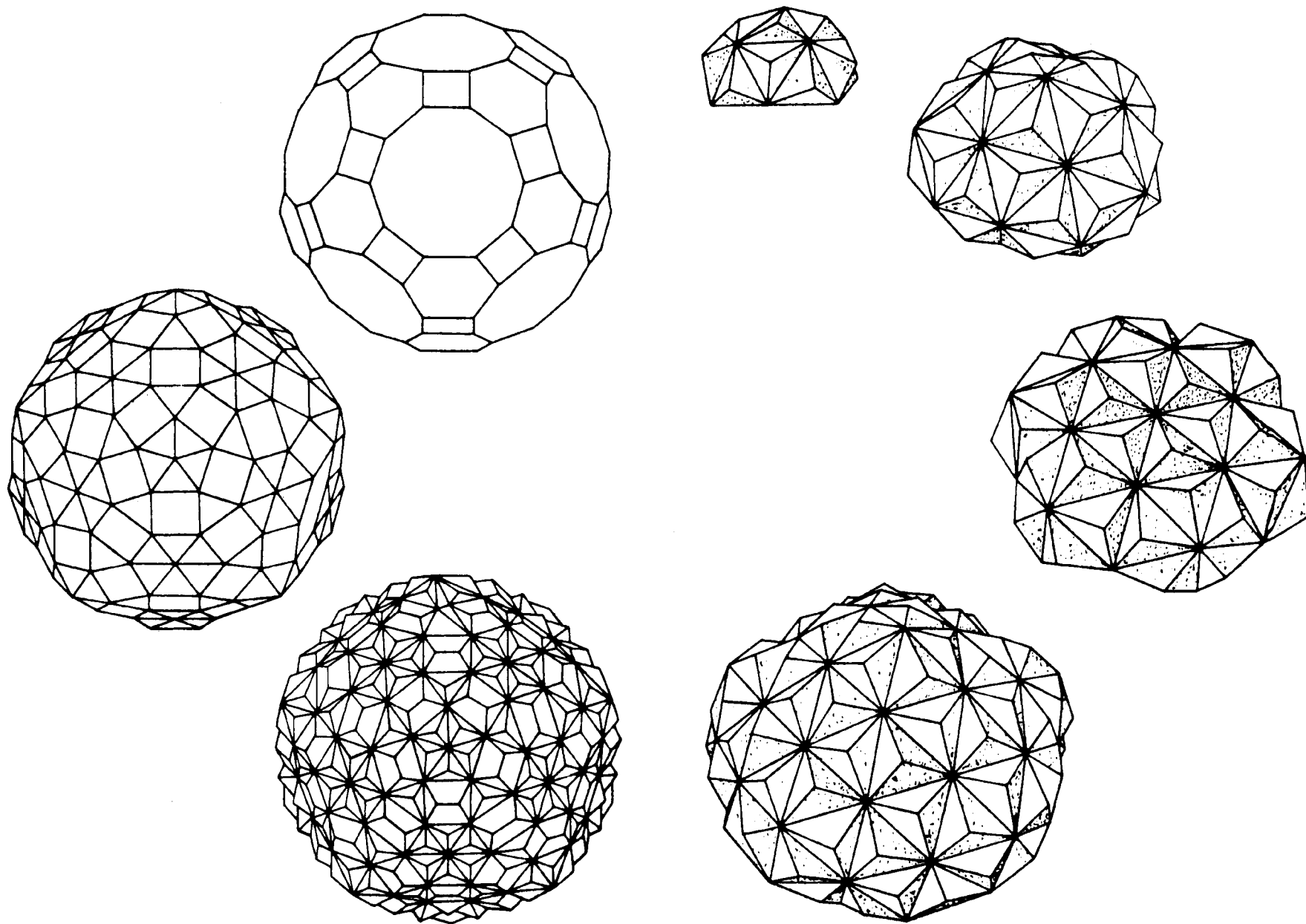


Figure 1 Des dômes stéréométriques • Some stereometric domes.

Dans ce qui suit, les divers polygones réguliers seront désignés par le chiffre correspondant respectivement au nombre d'arêtes formant leur périmètre; quant à la configuration sommitale caractéristique de chaque polyèdre, elle est définie chaque fois par le symbole Schläfli, qui consiste à énumérer dans un ordre cyclique les divers polygones se rencontrant à un sommet. Ainsi à titre d'exemple, 3465 signifie que le sommet est composé successivement d'un triangle, d'un carré, d'un hexagone et d'un pentagone, l'ordre cyclique commençant par le plus petit polygone. À noter également que dans les tableaux récapitulatifs, les majuscules S.A.F. signifient respectivement le nombre de sommets, d'arêtes et de faces du polyèdre.

Les polyèdres réguliers et semi-réguliers peuvent être disséqués par clivage, c'est-à-dire par une troncature planaire passant seulement à travers des sommets et des arêtes. On obtient ainsi un certain nombre de polyèdres incomplets: les pyramides, les calottes, les demi-solides et les troncs (frusta). Ces derniers sont des parties contenant une portion plus grande que la moitié d'un polyèdre, et où l'angle dièdre formé avec la base est supérieur à 90° . Les bases des troncatures, c'est-à-dire les nouvelles faces obtenues par clivage, sont également des polygones réguliers de 3, 4, 5, 6, 8 et 10 côtés.

En faisant abstraction des troncs, mais en incluant le tétraèdre qui, n'ayant pas de plan de clivage autre que sa face, est sa propre troncature, il existe sept troncatures régulières où l'angle dièdre formé avec la base est inférieur à 90° . Elles sont représentées sur la **Figure 2** et leurs caractéristiques morphologiques et métriques sont récapitulées au **Tableau 1**.

L'accollage par leurs bases des troncatures identiques donne des bi-pyramides ou des bi-calottes en forme de lentille. L'association des troncatures ou des troncs avec les prismes ou les antiprismes de base correspondante, produit un grand nombre de polyèdres hybrides convexes et un nombre pratiquement illimité de configurations hybrides non convexes, de forme générale oblongue, arborescente, toroïde ou spongieuse. Les polyèdres composites réguliers sont obtenus par une hybridation régulière.

Les solides réguliers composites topologiquement convexes sont composés par l'assemblage systématique de polyèdres réguliers ou semi-réguliers et de troncatures. En appliquant de façon homogène les troncatures par leur base sur toutes les faces homothétiques du polyèdre de forme correspondante à celle des bases, on obtient un nombre déterminé de solides métriquement non convexes ayant des angles dièdres alternativement saillants et rentrants, ces derniers égaux ou supérieurs à 180° .

L'enveloppe des polyèdres composites, dans la plupart des cas indéveloppables, comporte donc, outre les assemblages convexes, des assemblages non convexes où la somme des angles polygonaux varie entre 360° et 630° , étant composés de quatre à dix polygones réguliers. Ces assemblages non convexes sont énumérés par catégories, colonnes **P4**, ..., **P10**, suivant le nombre de polygones formant les assemblages, au **Tableau 2**.

Les sommets d'un polyèdre composite se situent sur deux ou plusieurs sphères concentriques, suivant qu'on utilise 1, 2 ou 3 sortes de troncatures comme complément et que les sommets de ces troncatures s'étagent sur 1 ou 2 niveaux.

In what follows, the various regular polygons are labelled with the number of their edges, and the characteristic vertex configuration of each polyhedron is defined by the Schläfli symbol, which enumerates in a cyclic order the various polygons which meet in a vertex. For example, 3465 means that the vertex is successively composed of a triangle, a square, a hexagon and a pentagon, with the cyclic order starting at the smallest polygon. Note that the acronym V.E.F. used in the tables stands for the respective number of vertices, edges and faces.

The regular and semi-regular polyhedra can be dissected by truncating them, i.e. by cutting them with a plane passing through vertices and edges only. Thus we obtain a number of incomplete polyhedra: pyramids, calottes, semi-solids and frusta. The latter are pieces which contain more than half of the cut polyhedron, and their dihedral angle formed with the base-plane is more than 90° . The bases of the pieces which are cut, i.e. the new faces created by the cutting process, are also regular polygons with 3, 4, 5, 6, 8 and 10 sides.

Without considering the frusta, but including the tetrahedra where the base is the cutting face, there are seven regular truncations in which the dihedral angle of the base is less than 90° . These truncations are pictured in **Figure 2** and their morphologic and metric characteristics are summarized in **Table 1**.

Attaching identical truncations by pasting together their bases, gives bi-pyramids or bi-calottes in form of a lens. By associating truncations of frusta with prisms or antiprisms with a corresponding base, one can produce a large number of hybrid convex polyhedra and a practically unlimited number of hybrid non-convex configurations, of general form: oblong, arborescent, toroidal or spongy. The regular composite polyhedra have been obtained by a regular hybridization.

The topologically convex regular composite solids are composed by systematically assembling regular or semi-regular polyhedra and truncations. By attaching, in a homogeneous manner, the truncations by their base to all homothetic faces of a polyhedron the same form as this base, we obtain a fixed number of solids which are metrically non-convex and have dihedral angles salient and re-entering alternatively, with the latter ones greater or equal to 180° .

The envelope of a composite polyhedron, in the majority of cases not developable, consists not only of convex compositions but also of non-convex ones where the sum of the polygonal angles varies between 360° and 630° , and are composed of four to ten regular polygons. The non-convex compositions are enumerated by category, columns **P4**, ..., **P10**, following the number of polygons which form these compositions, in **Table 2**.

The vertices of a composite polyhedron lie on two or more concentric spheres; this means that we use 1, 2 or 3 kinds of truncations as complements and that the vertices of those truncations lie on 1 or 2 levels.

Les opérations peuvent être exécutées intégralement, couvrant la totalité des faces avec des compléments transplantés: cas des polyèdres composites saturés; ou seulement partiellement, sur les faces polygonales de même type, les autres faces étant conservées: cas des solides non saturés. Les troncatures étant de deux catégories, les pyramides et les calottes, les polyèdres composites peuvent être classés en trois classes suivant qu'ils sont produits par pyramidation, par plissement à l'aide des calottes ou par les deux opérations à la fois.

Cent deux polyèdres composites peuvent être construits par hybridation régulière, chacune des trois classes contenant 34 solides présentés par des figures schématiques et des photographies des modèles correspondants (Figures 3, 4 et 5).

Dans les Tableaux 3, 4 et 5, les codes Schläfli des trois classes de polyèdres composites seront énumérés dans les colonnes n1, n2, n3 et n4, leurs configurations sommitales étant de 1, 2, 3 ou 4 sortes. De même leur enveloppe étant composée de diverses faces polygonales de 3, 4, 5, 6, 8 et 10 côtés, on trouvera le sous-total de chaque sorte de faces respectivement dans les colonnes f3, f4, f5, f6, f8 et f10.

The operations may be carried out completely, thus covering all faces with transplanted complements: this is the case of saturated composite polyhedra; they may also be partially executed on polygonal faces of the same type whereas other faces remain as they are: this is the case of non-saturated solids. The truncations used are of two kinds: the pyramids and the calottes; the composite polyhedra may be classified in three classes depending on whether they are produced by stellation, by corrugation using calottes, or by both operations at the same time.

A hundred and two composite polyhedra can be constructed by regular hybridization, since each of the three classes contains 34 solides which are presented by figures and by photographs of corresponding models (Figures 3, 4 and 5).

In Tables 3, 4 and 5, the Schläfli symbols of the three classes of composite polyhedra appear in columns n1, n2, n3, n4, since their vertex configuration can be of 1, 2, 3 or 4 kinds. Similarly, the subtotal for each kind of faces of the envelope which is composed of various polygonal faces of 3, 4, 5, 6, 8 and 10 sides, is indicated in columns f3, f4, f5, f6, f8 and f10 respectively.

Tableau 1 — Table 1
Les troncatures régulières The regular truncations

nom	code des troncatures	polygone de la base	S	A	nombre de faces par type de polygone						F	a		
					f3	f4	f5	f6	f8	f10				
1 pyramide triangulaire (tétraèdre)	333	3	4	6	4						4	3-3	70°32'	triangular pyramid (tetrahedron) 1
2 pyramide carrée (demi-octaèdre)	3333	4	5	8	4	1					5	3-4	54°44'	square pyramid (half-octahedron) 2
3 pyramide pentagonale (calotte icosaédrique)	33333	5	6	10	5		1				6	3-5	37°23'	pentagonal pyramid (icosahedral calotte) 3
4 demi-cuboctaèdre	3434	6	9	15	4	3		1			8	3-6	70°32'	half-cuboctahedron 4
												4-6	54°44'	
5 calotte rhombicuboctaédrique	3444	8	12	20	4	5			1		10	3-8	54°44'	rhombicuboctahedral calotte 5
												4-8	45°	
6 calotte rhombicosidodécaédrique	3454	10	15	25	5	5	1			1	12	3-10	37°23'	rhombicosidodecahedral calotte 6
												4-10	31°43'	
7 demi-icosidodécaèdre	3535	10	20	35	10		6			1	17	3-10	79°11'	half-icosidodecahedron 7
												5-10	63°26'	
	code of truncations	polygon of base	V	E	f3	f4	f5	f6	f8	f10	F	a		name
					number of faces by type of polygon									

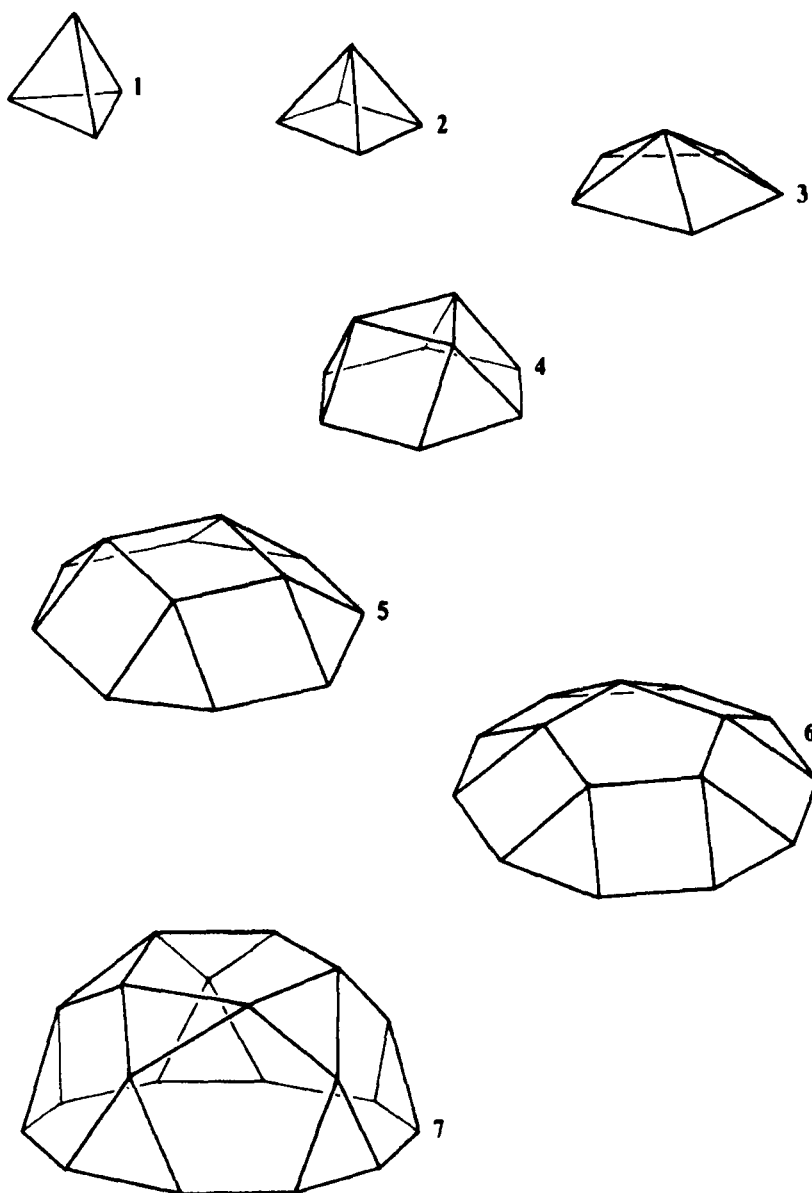


Tableau 2 — Table 2
Les assemblages non convexes The non-convex compositions

P4	P5	P6	P7	P8	P9	P10
3 4 4 6	3 3 3 4 4	3 3 3 3 3 3	3 ⁷	3 ⁸	3 ⁸ 5	3 ¹⁰
3 4 6 4	3 3 4 3 4	3 3 3 3 4 4	3 ⁶ 5			3 ⁹ 4
3 4 6 5	3 3 4 4 4	3 3 3 4 3 4				
3 5 4 6	3 3 4 4 5	3 3 4 3 3 4				
3 3 6 8	3 3 4 5 4	3 3 3 3 3 5				
3 4 4 8	3 4 3 4 5	3 3 3 4 3 5				
3 4 8 4	3 4 4 3 5	3 3 4 3 3 5				
3 3 6 10	3 3 3 5 5	3 3 3 3 5 5				
3 4 4 10	3 3 5 3 5	3 3 5 3 3 5				
3 4 10 4	3 3 4 3 6					
3 3 10 10	3 3 5 3 6					
	3 3 3 4 8					
	3 3 4 3 8					
	3 3 4 3 10					

Figure 2 Les sept troncatures régulières • The seven regular truncations.

Tableau 3 — Table 3
Les polyèdres pyramidés The stellated polyhedra

n°	base	nombre de pyramides			code des solides pyramidés				nombre de faces par type de polygone						total		
		3 ³	3 ⁴	3 ⁵	n1	n2	n3	n4	f3	f4	f5	f6	f8	f10	F	A	S
1	3 3 3	4			3 ⁶	3 ³			12						12	18	8
2	3 3 3 3	8			3 ⁸	3 ³			24						24	36	14
3	3 3 3 3 3	20			3 ¹⁰	3 ³			60						60	90	32
4	4 4 4		6		3 ⁶	3 ⁴			24						24	36	14
5	5 5 5			12	3 ⁶	3 ⁵			60						60	90	32
6	3 3 3 3 4	32			3 ⁸ 4	3 ³			96	6					102	156	56
7			6		3 ⁶	3 ⁴			56						56	84	30
8		32	6		3 ¹⁰	3 ⁴	3 ³		120						120	180	62
9	3 4 3 4	8			3 ³	3 ² 4 3 ² 4			24	6					30	48	20
10			6		3 ⁶	3 ⁴			32						32	48	14
11		8	6		3 ⁸	3 ⁴	3 ³		48						48	72	26
12	3 4 4 4	8			3 ² 4 ³	3 ³			24	18					42	72	32
13			18		3 ⁷	3 ⁴			80						80	120	42
14		8	18		3 ⁸	3 ⁴	3 ³		96						96	144	50
15	3 3 3 3 5	80			3 ⁸ 5	3 ³			240		12				252	390	140
16				12	3 ⁶	3 ⁵			140						140	210	72
17		80		12	3 ¹⁰	3 ⁵	3 ³		300						300	450	152
18	3 4 5 4	20			3 ² 4 5 4	3 ³			60	30	12				102	180	80
19			30		3 ⁵ 5	3 ⁴			110		12				122	210	90
20				12	3 ² 4 3 4	3 ⁵			80	30					110	180	72
21		20	30		3 ⁶ 5	3 ⁴	3 ³		180		12				192	300	110
22		20		12	3 ² 4 3 ² 4	3 ⁵	3 ³		120	30					150	240	92
23			30	12	3 ⁷	3 ⁵	3 ⁴		200						200	300	102
24		20	30	12	3 ⁸	3 ⁵	3 ⁴	3 ³	240						240	360	122
25	3 5 3 5	20			3 ² 5 3 ² 5	3 ³			60		12				72	120	50
26				12	3 ³	3 ² 6 ²			80						80	120	42
27		20		12	3 ⁸	3 ⁵	3 ³		120						120	180	62
28	3 6 6	4			3 ³	3 ² 6 ²			12			4			16	30	16
29	4 6 6		6		3 ⁴	3 ² 6 ²			24			8			32	60	30
30	5 6 6			12	3 ⁵	3 ² 6 ²			60			20			80	150	152
31	4 6 8		12		3 ² 6 8	3 ⁴			48			8	6		62	120	60
32	3 8 8	8			3 ² 8 ²	3 ³			24				6		30	60	32
33	4 6 10		30		3 ² 6 10	3 ⁴			120			20		12	152	300	150
34	3 10 10	20			3 ² 10 ²	3 ³			60					12	72	150	80
n°	base	3 ³	3 ⁴	3 ⁵	n1	n2	n3	n4	f3	f4	f5	f6	f8	f10	F	E	V
		number of pyramids			code of stellated solids				number of faces by type of polygon						total		

Polyèdres pyramidés

Les polyèdres pyramidés sont obtenus en complétant les solides platoniques ou archimédiens par un ou plusieurs types de pyramides régulières, de base triangulaire, carrée ou pentagonale appliquées sur les faces polyédriques correspondantes. Quand les polyèdres de support originalement composés de faces de 3, 4 ou 5 côtés seulement, sont saturés par une pyramidation intégrale, on obtient des solides entièrement et équilatéralement triangulés, dont le plus grand est composé de 300 faces. En reliant les sommets saillants vers l'extérieur, ces polyèdres entièrement pyramidés sont facilement transformables en autant de réseaux à double nappe courbes. Les caractéristiques de ces solides sont résumées au **Tableau 3**.



Figure 3.1

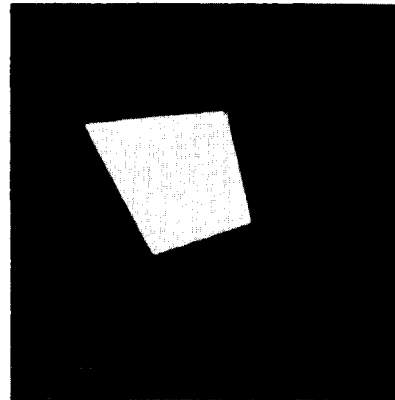


Figure 3.2

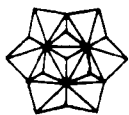
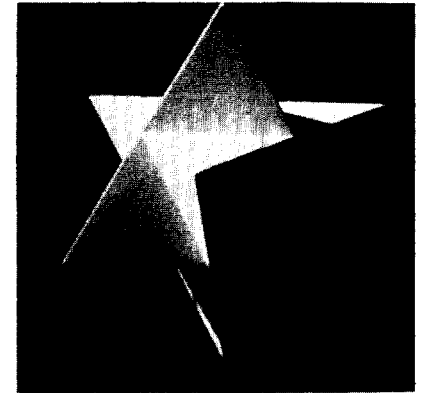


Figure 3.3

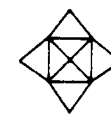
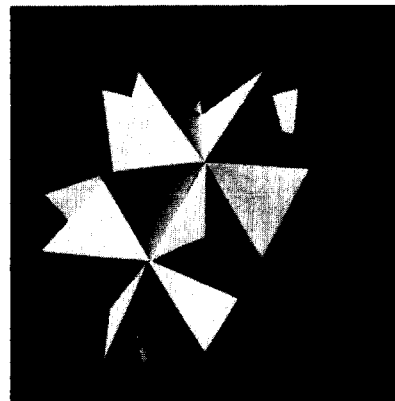
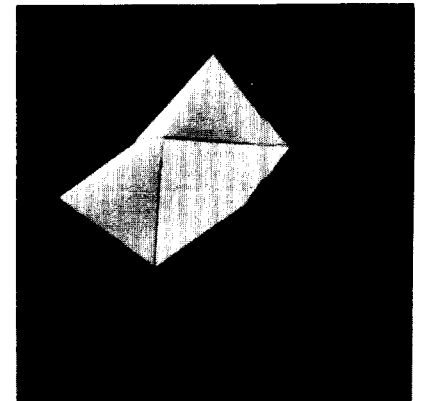


Figure 3.4



Stellated Polyhedra

Stellated polyhedra are obtained by completing Platonic or Archimedean solids using various types of regular pyramids — of triangular, square or pentagonal base — applied on the corresponding faces of the polyhedra (solids). When the underlying polyhedra, originally containing faces with 3, 4 or 5 sides only, get saturated by this stellation process, then we obtain solids which are entirely covered with equilateral triangles, of which the largest has 300 faces. By joining their salient vertices, those completely stellated polyhedra can be easily transformed into curved two layer networks. The characteristics of those solids are summarized in **Table 3**.

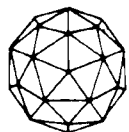


Figure 3.5

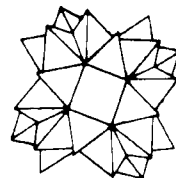
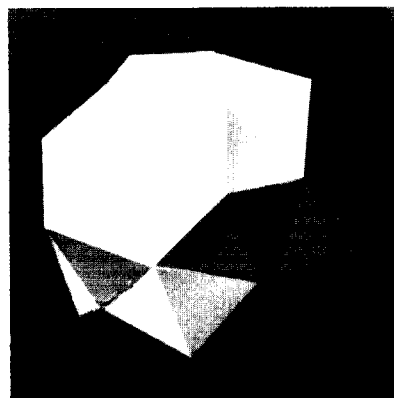


Figure 3.6

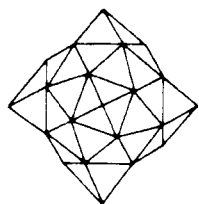
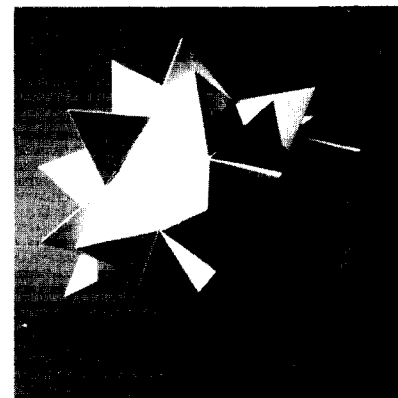


Figure 3.7

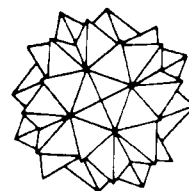
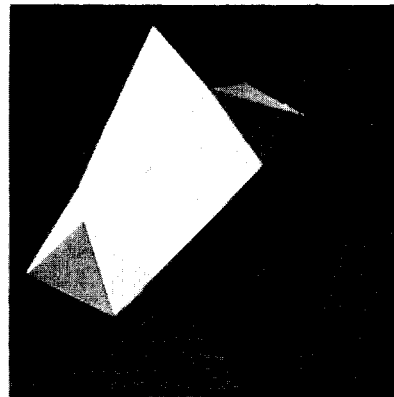


Figure 3.8

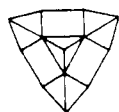
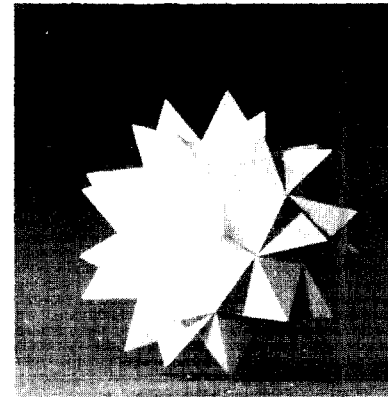


Figure 3.9

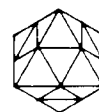
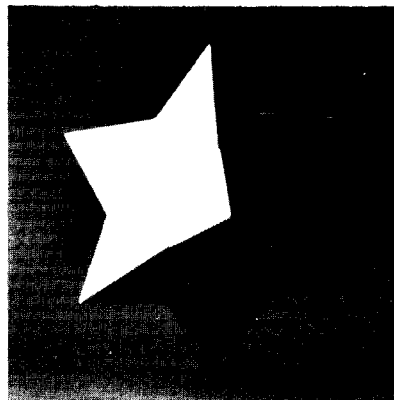
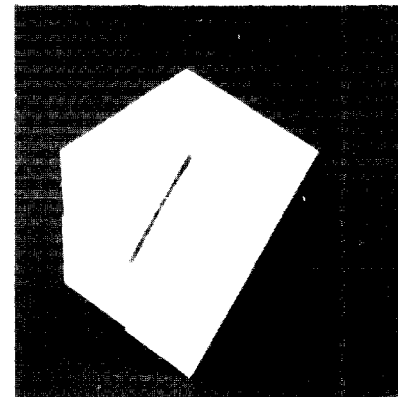


Figure 3.10



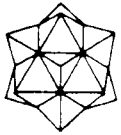


Figure 3.11

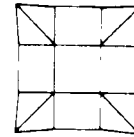
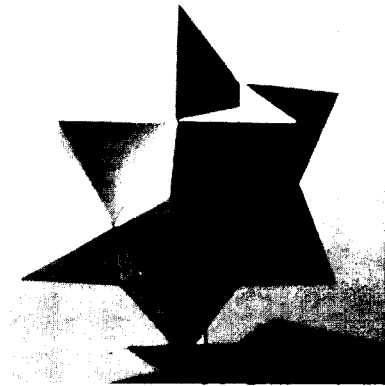


Figure 3.12

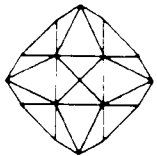
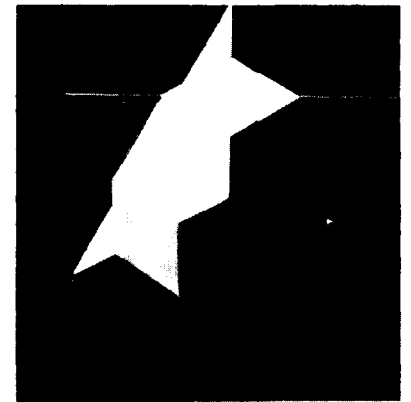


Figure 3.13

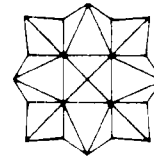
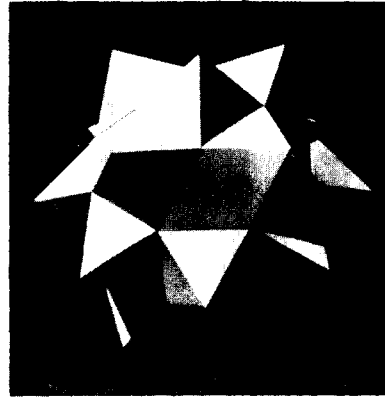


Figure 3.14

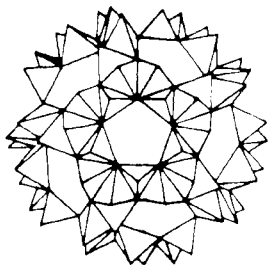
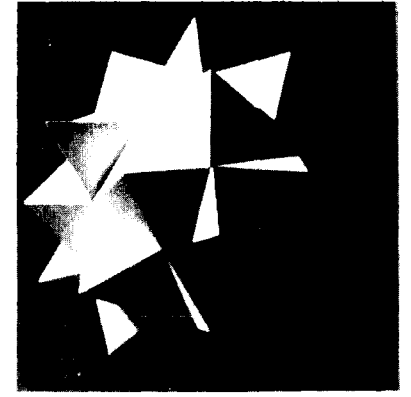


Figure 3.15

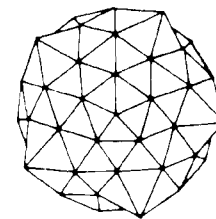
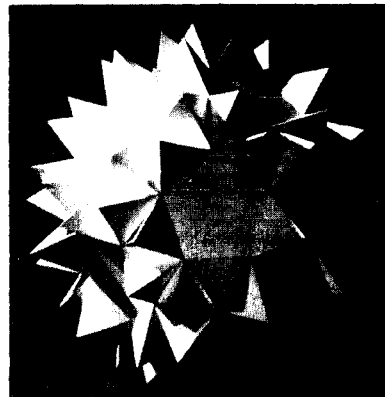
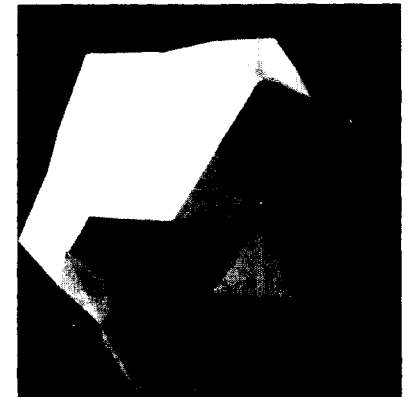


Figure 3.16



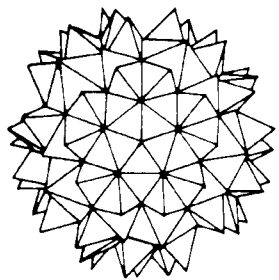


Figure 3.17

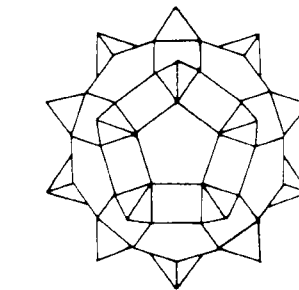
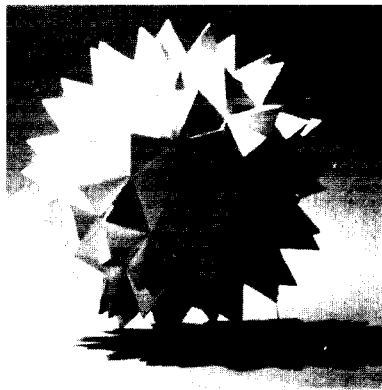


Figure 3.18

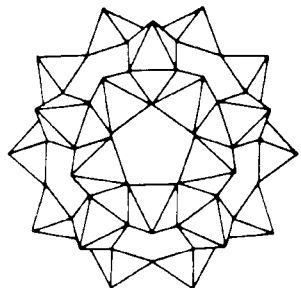
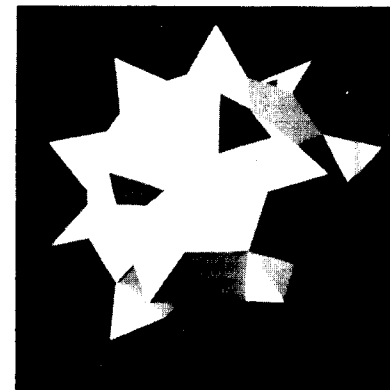


Figure 3.19

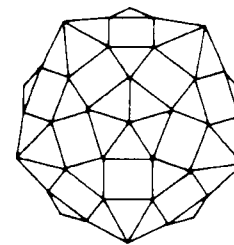
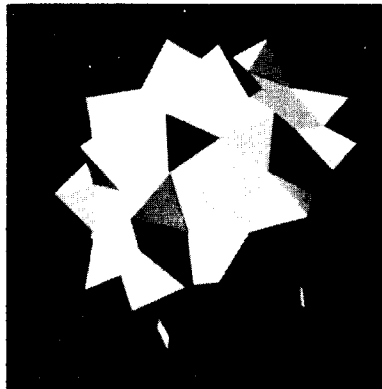


Figure 3.20

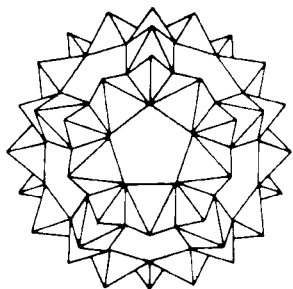
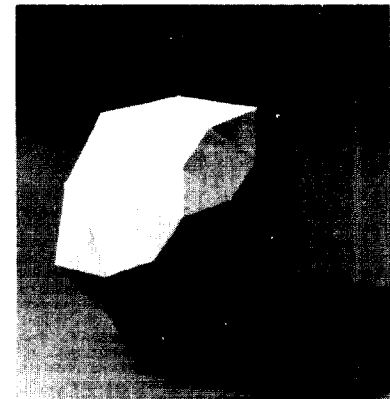


Figure 3.21

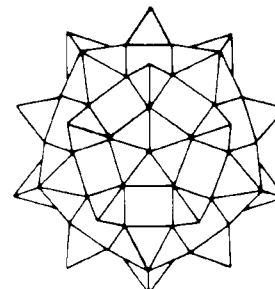
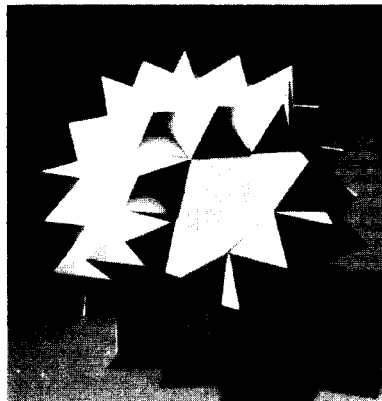
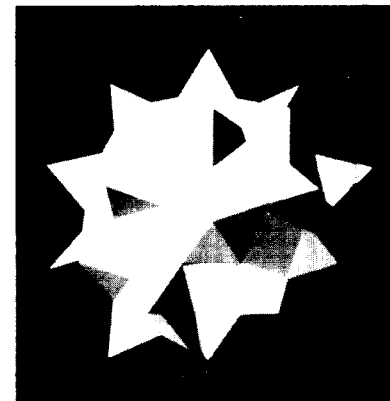


Figure 3.22



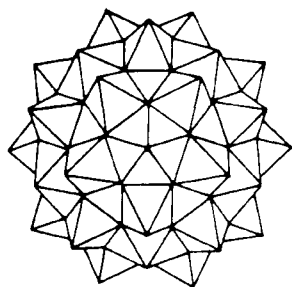


Figure 3.23

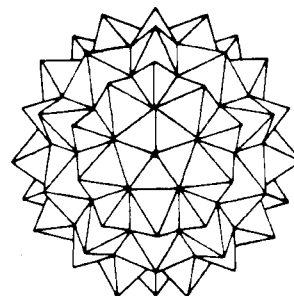
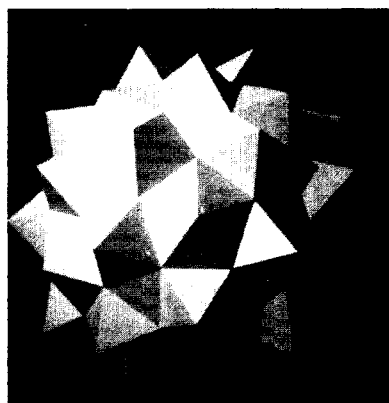


Figure 3.24

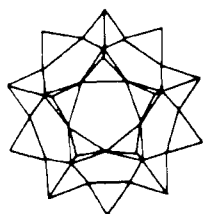
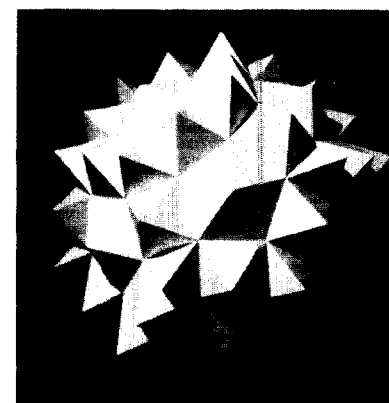


Figure 3.25

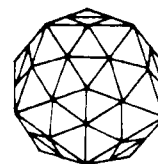
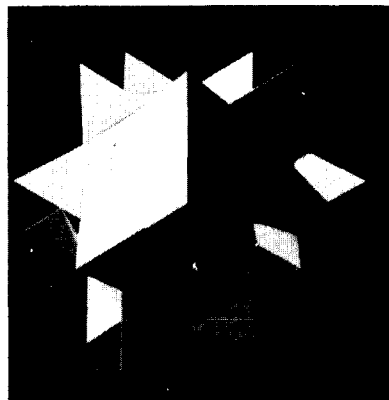


Figure 3.26

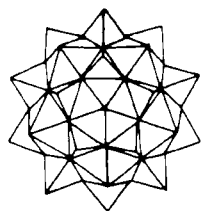
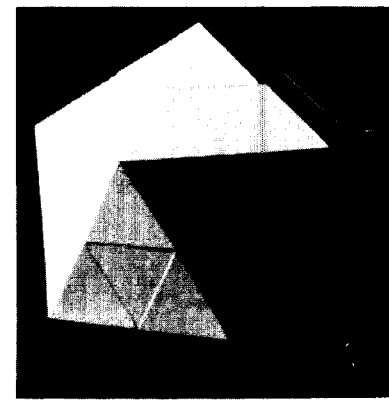


Figure 3.27

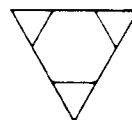
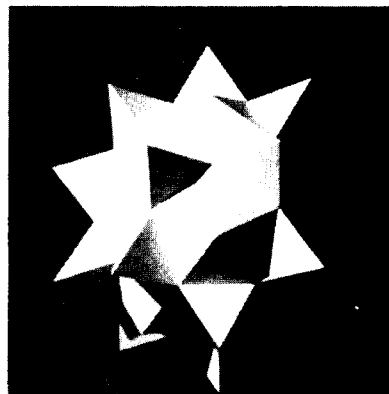
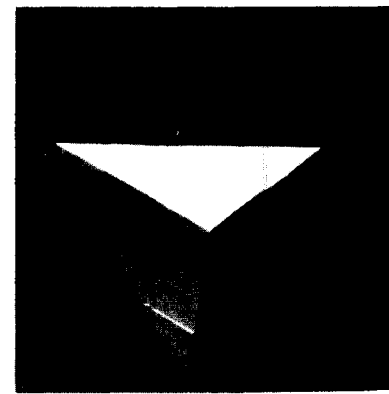


Figure 3.28



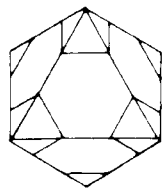


Figure 3.29

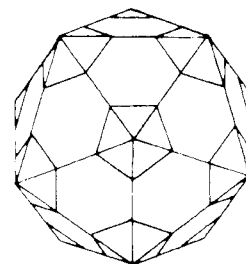
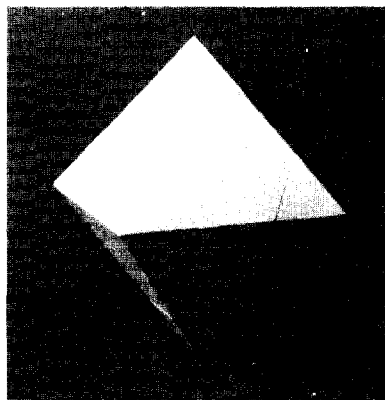


Figure 3.30

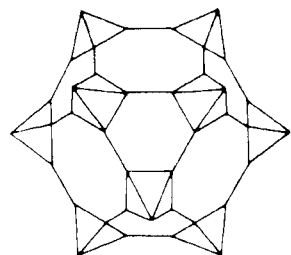
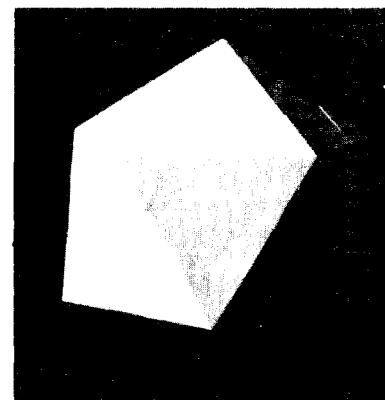


Figure 3.31

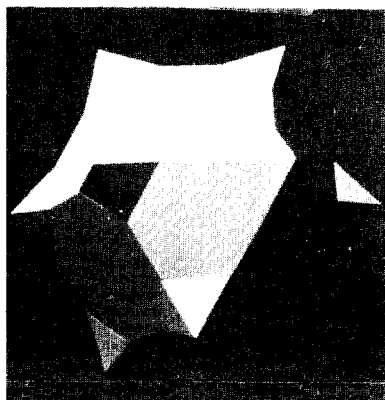


Figure 3.32

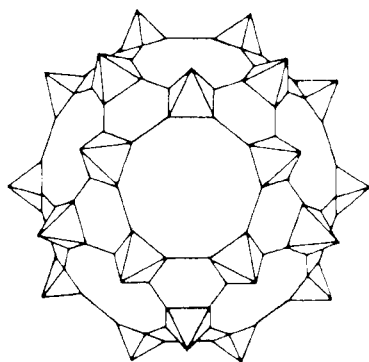
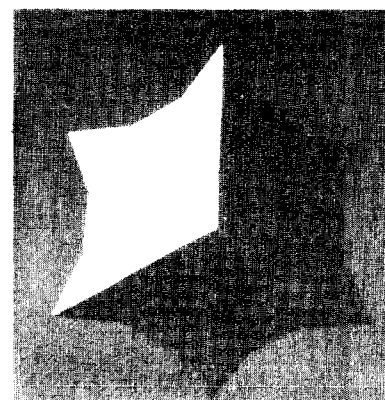


Figure 3.33

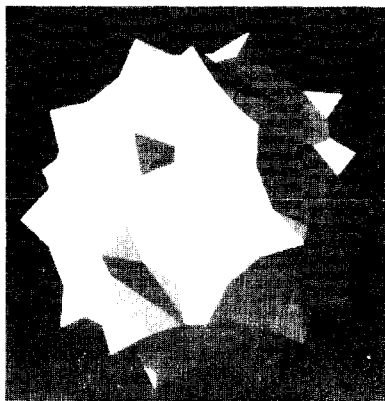
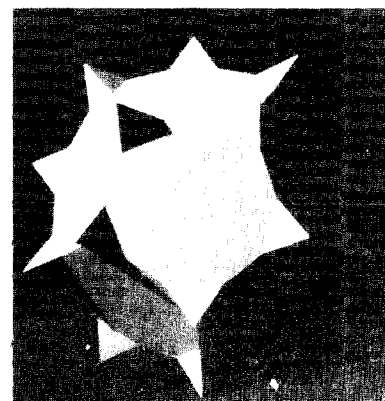


Figure 3.34



n°	base	troncatures				code des solides composites			nombre de faces par type de polygone						total		
		3434	3444	3454	3535	n1	n2	n3	f3	f4	f5	f6	f8	f10	F	A	S
1	366	4				33344	3434		20	12					32	54	24
2		4				33434	3434		20	12					32	54	24
3	466	8				3434	34344		32	30					62	108	48
4		8				33444	3434		32	30					62	108	48
5	566	20				3434	34435		80	60	12				152	270	120
6		20				33454	3434		80	60	12				152	270	120
7	468	8				3434	3484		32	36			6		74	144	72
8		8				3434	3448		32	36			6		74	144	72
9			6			3444	3446		24	42		8			74	144	72
10			6			3444	3464		24	42		8			74	144	72
11		8	6			33444	3434	3444	56	66					122	216	96
12		8	6			3434	34344	3444	56	66					122	216	96
13		8	6			3434	34344	3444	56	66					122	216	96
14		8	6			3434	34344	3444	56	66					122	216	96
15	388		6			33434	3444		32	30					62	108	48
16			6			33344	3444		32	30					62	108	48
17	4610	20				3434	34104		80	90				12	182	360	180
18		20				3434	34410		80	90				12	182	360	180
19				12		3454	3464		60	90	12	20			182	360	180
20				12		3454	3446		60	90	12	20			182	360	180
21					12	3535	3465		120	30	72	20			242	480	240
22					12	3535	3546		120	30	72	20			242	480	240
23		20		12		33444	3434	3454	140	150	12				302	540	240
24		20		12		3434	34344	3454	140	150	12				302	540	240
25		20		12		3434	34344	3454	140	150	12				302	540	240
26		20		12		3434	34344	3454	140	150	12				302	540	240
27		20			12	33445	3434	3535	200	90	72				362	660	300
28		20			12	3434	34345	3535	200	90	72				362	660	300
29		20			12	3434	34435	3535	200	90	72				362	660	300
30		20			12	3434	34345	3535	200	90	72				362	660	300
31	31010			12		33344	3454		80	60	12				152	270	120
32				12		33434	3454		80	60	12				152	270	120
33					12	33355	3535		140		72				212	390	180
34					12	33535	3535		140		72				212	390	180
n°	base	3434	3444	3454	3535	n1	n2	n3	f3	f4	f5	f6	f8	f10	F	E	V
		truncations				code of composite solids			number of faces by type of polygon						total		

Polyèdres plissés

Les polyèdres plissés sont obtenus en complétant les solides semi-réguliers par un ou plusieurs types de calottes appliquées sur les faces hexagonales, octogonales ou décagonales du solide de base. Ces polyèdres présentent des lobes multiples ou plissements. Les solides, dont les faces sont saturées par transplantation de calottes, sont composés uniquement de faces triangulaires, carrées et pentagonales. Leurs caractéristiques sont rapportées au **Tableau 4**.

Corrugated Polyhedra

Corrugated polyhedra are obtained by completing semi-regular solids with calottes of one or more types, applied on hexagonal, octagonal or decagonal faces of the underlying solid. These polyhedra contain multiple lobes or corrugations. The solids whose faces are saturated by transplantation of calottes are composed of triangular, square and pentagonal faces only. Their characteristics are reported in **Table 4**.

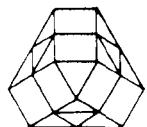


Figure 4.1

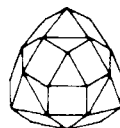
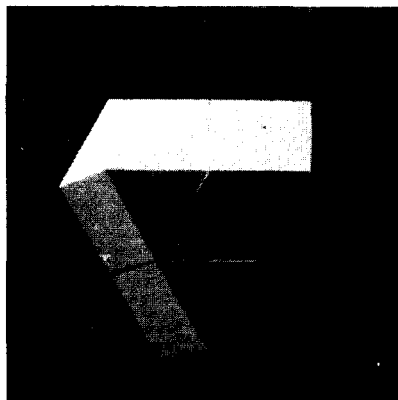


Figure 4.2

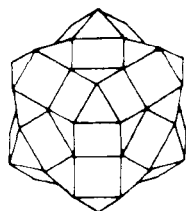
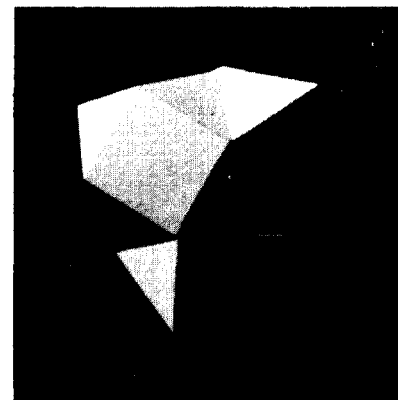


Figure 4.3

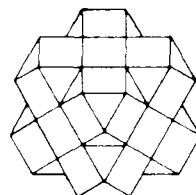
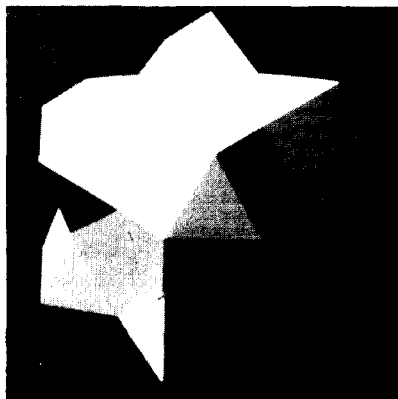
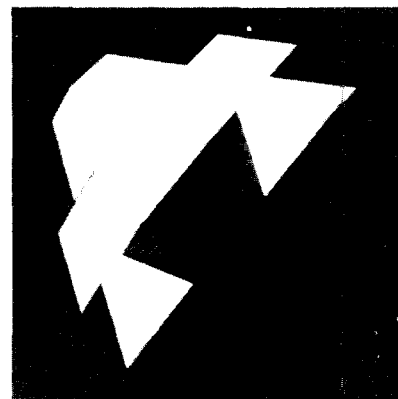


Figure 4.4



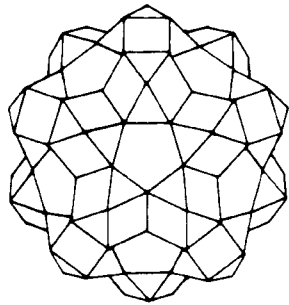


Figure 4.5

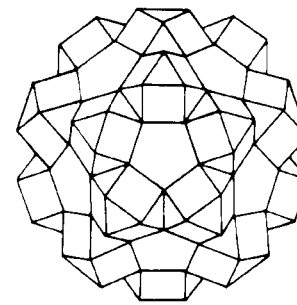
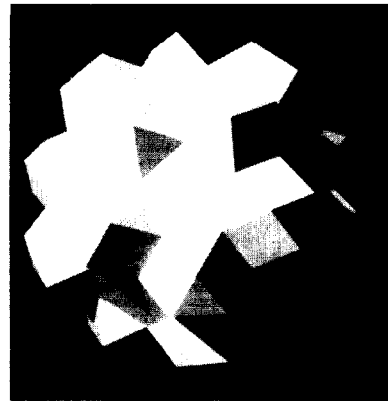


Figure 4.6

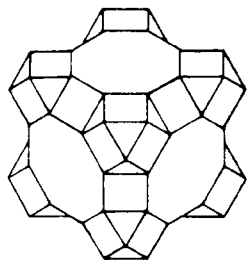
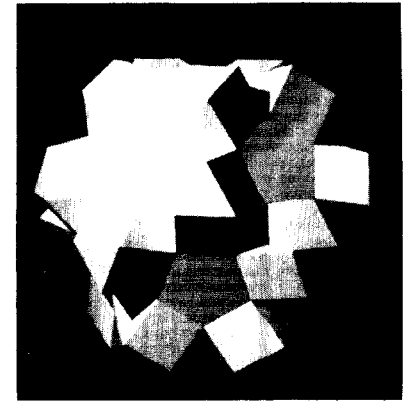


Figure 4.7

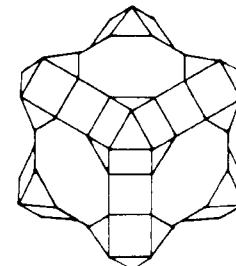
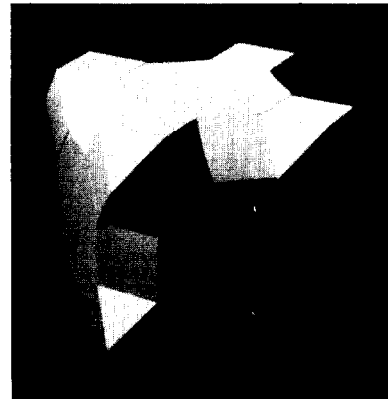


Figure 4.8

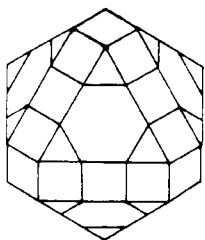
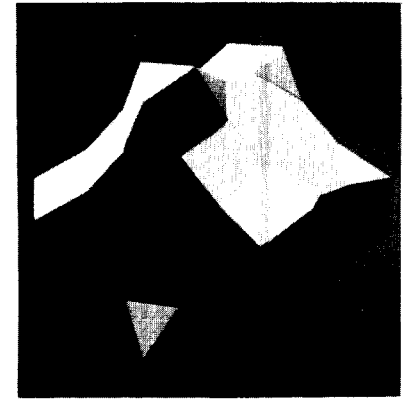


Figure 4.9

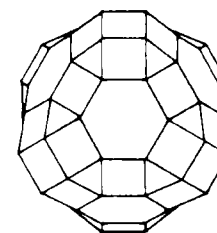
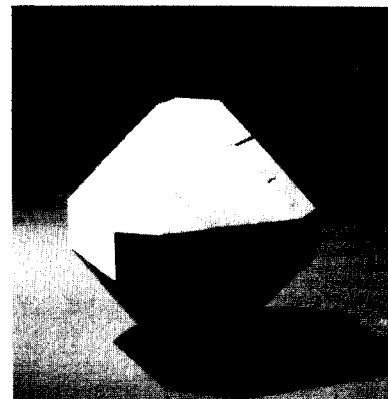
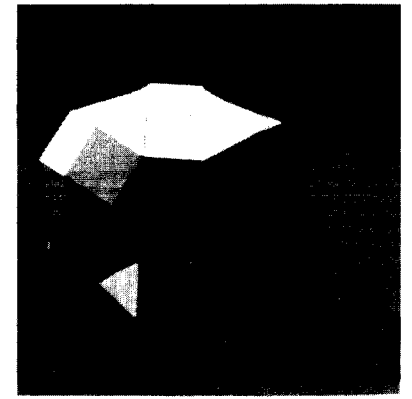


Figure 4.10



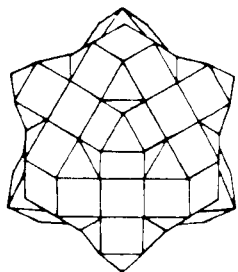


Figure 4.11

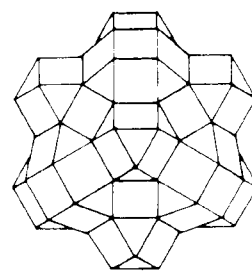
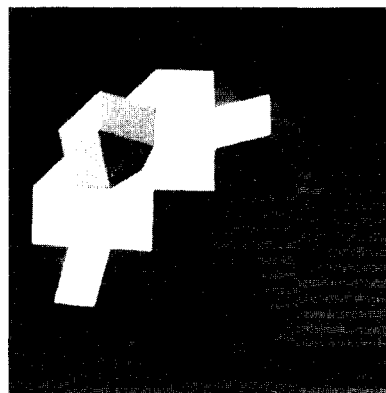


Figure 4.12

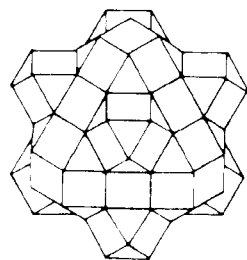
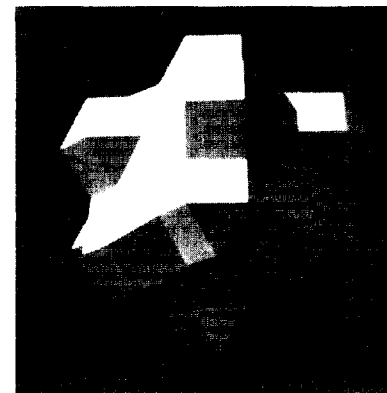


Figure 4.13

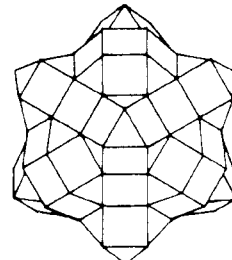
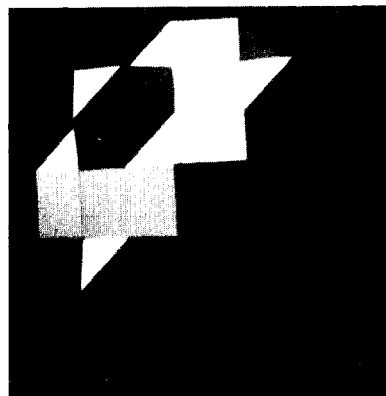


Figure 4.14

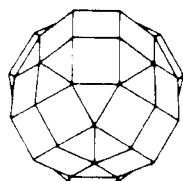
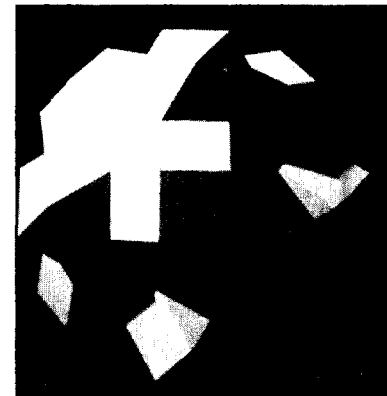


Figure 4.15

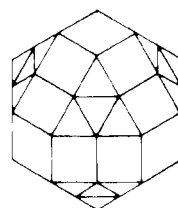
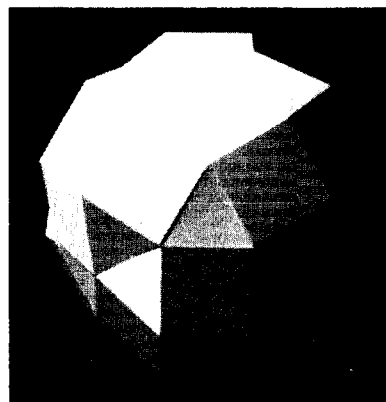
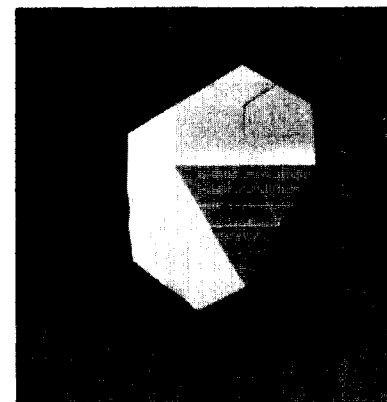


Figure 4.16



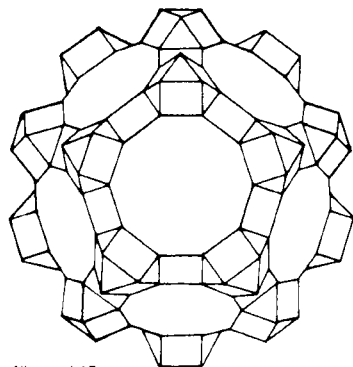


Figure 4.17

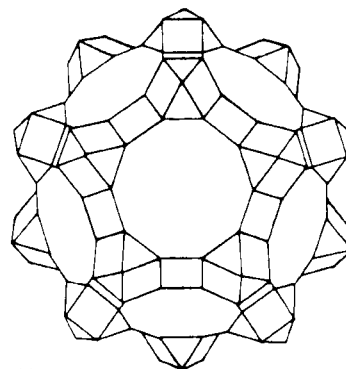
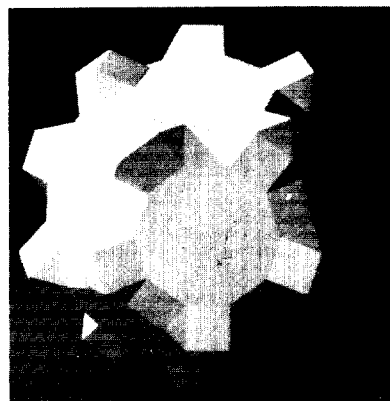


Figure 4.18

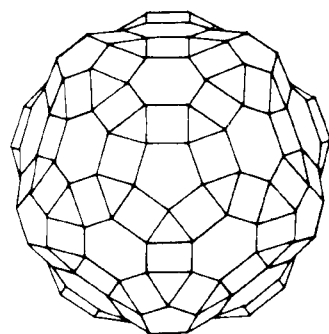
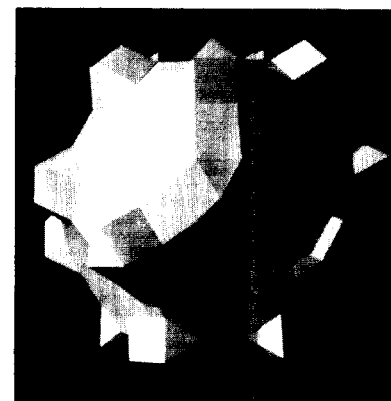


Figure 4.19

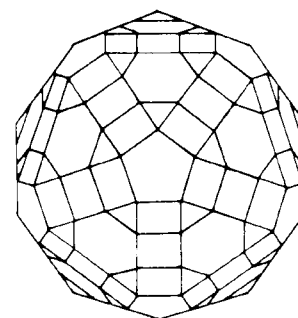
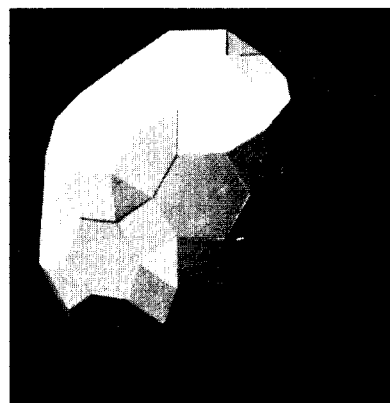


Figure 4.20

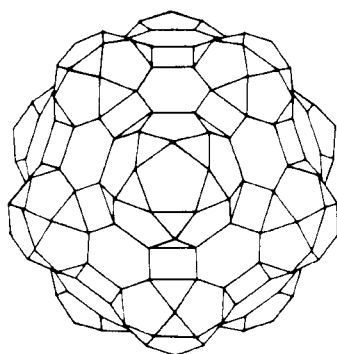
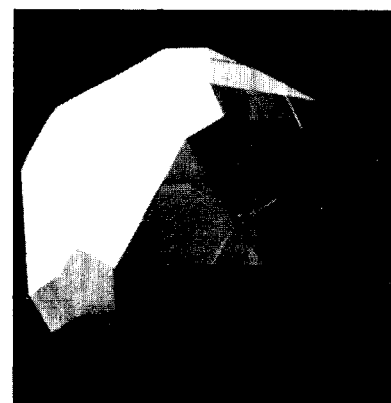


Figure 4.21

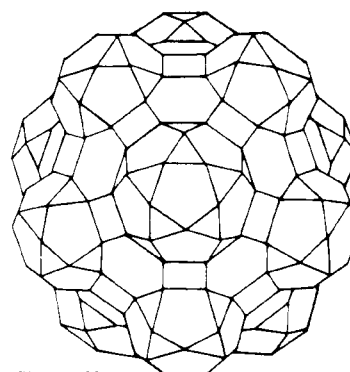
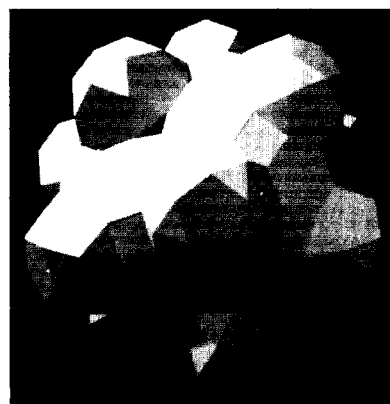


Figure 4.22



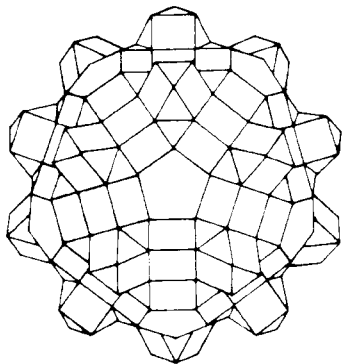


Figure 4.23

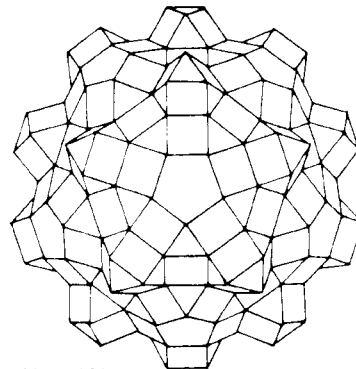
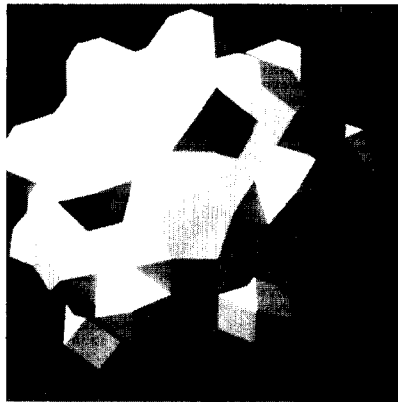


Figure 4.24

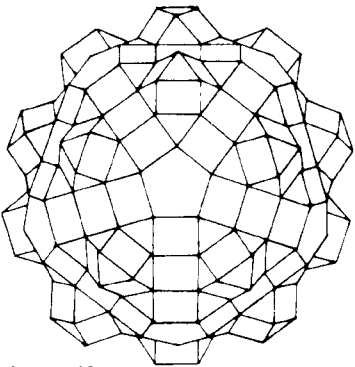
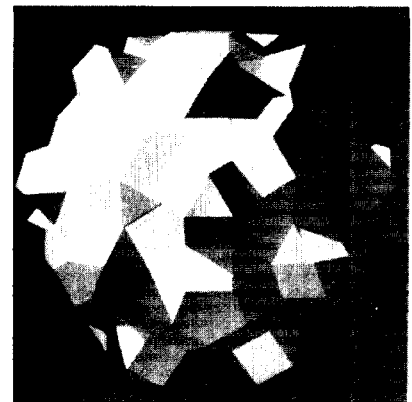


Figure 4.25

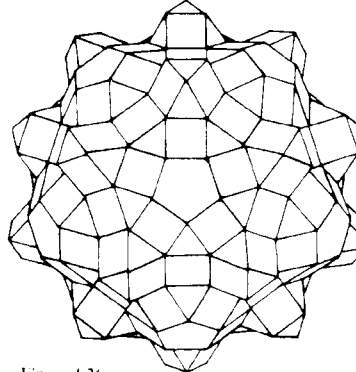
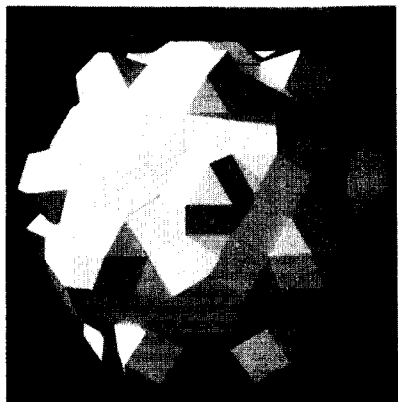


Figure 4.26

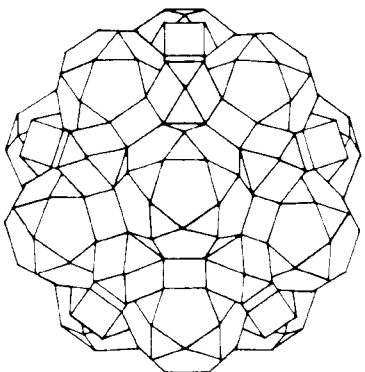
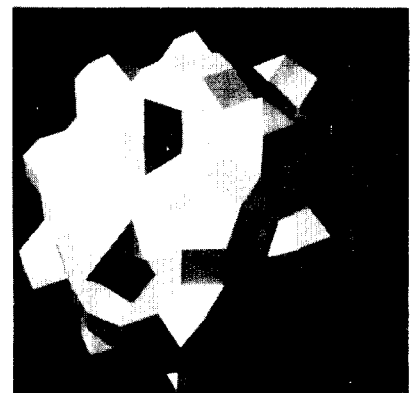


Figure 4.27

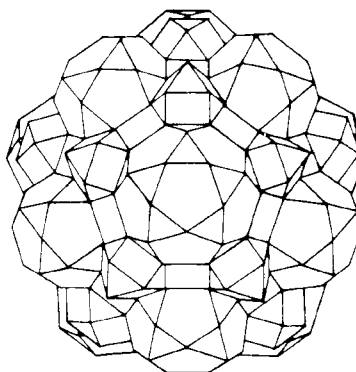
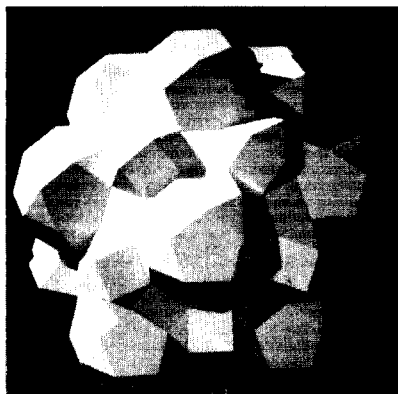
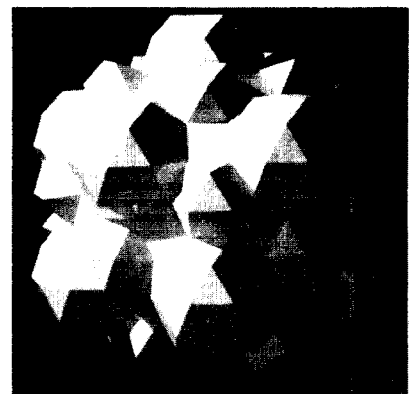


Figure 4.28



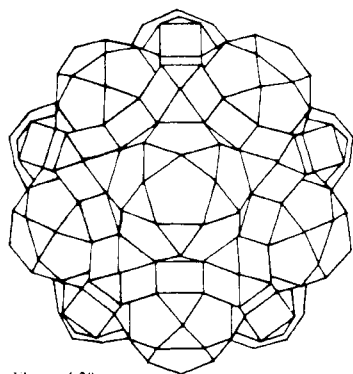


Figure 4.29

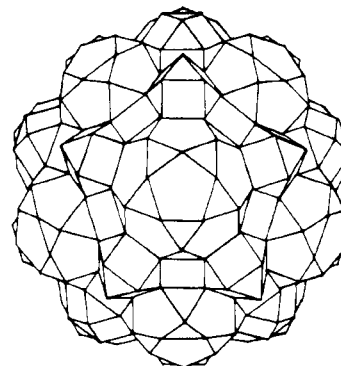


Figure 4.30

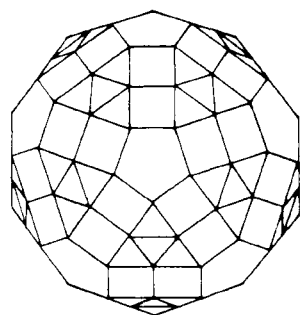
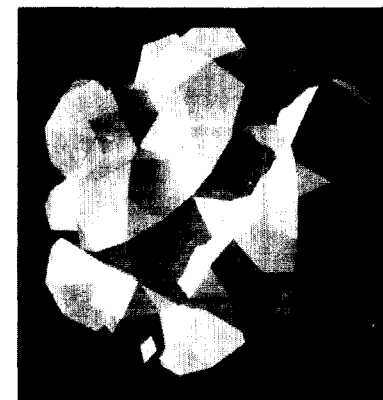


Figure 4.31

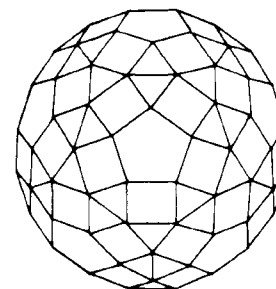
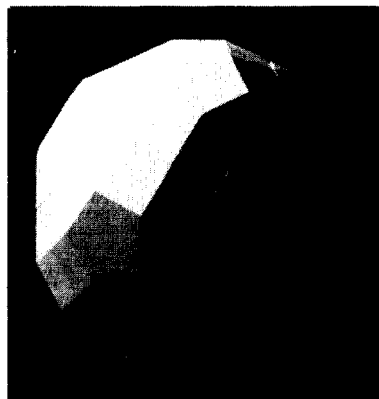


Figure 4.32

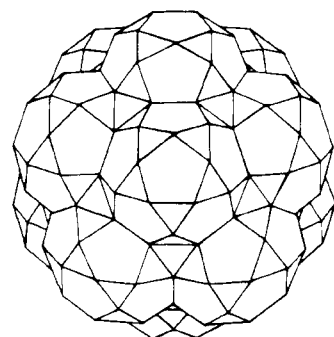
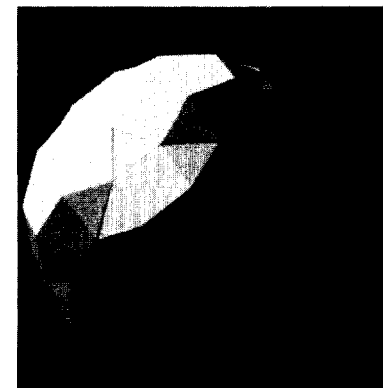


Figure 4.33

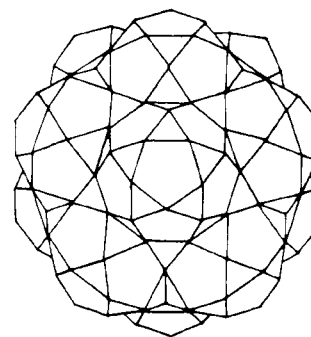
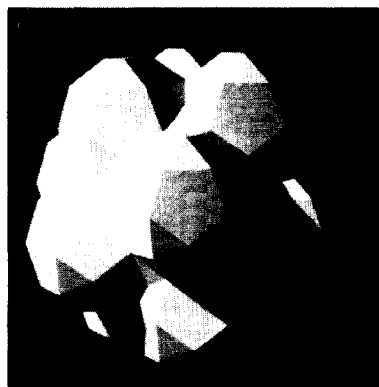


Figure 4.34

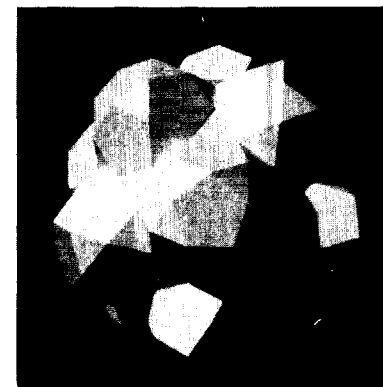


Tableau 5 — Table 5
Les polyèdres plissés et pyramidés The corrugated and stellated polyhedra

n°	base	polyèdres plissés			pyramides compléments			code des solides composites				nombre de faces par type de polygone						total		
					3 ³	3 ⁴	3 ⁵	n1	n2	n3	n4	f3	f4	f5	f6	f8	f10	F	A	S
1	366	33344	3434		4			333	333344	3434		28	12					40	66	28
2		33434	3434		4			333	334334	3434		28	12					40	66	28
3	466	3434	34344			6		3333	333344	3434		56	24					80	132	54
4		33444	3434			6		3333	334334	3434		56	24					80	132	54
5	566	3434	34435				12	33333	333344	3434		140	60					200	330	132
6		33454	3434				12	33333	334334	3434		140	60					200	330	132
7	468	3434	3484			12		3333	33348	3434		80	24			6		110	192	84
8		3434	3448			12		3333	33438	3434		80	24			6		110	192	84
9		3444	3446			12		3333	33436	3444		72	30		8			110	192	84
10		3444	3464			12		3333	33346	3444		72	30		8			110	192	84
11		33444	3434	3444		12		3333	334334	3434	3444	104	54					158	264	108
12		3434	34344	3444		12		3333	333344	3434	3444	104	54					158	264	108
13		3434	34344	3444		12		3333	333434	3434	3444	104	54					158	264	108
14		3434	34344	3444		12		3333	333434	3434	3444	104	54					158	264	108
15	388	33434	3444		8			333	334334	3444		48	30					78	132	56
16		33344	3444		8			333	333344	3444		48	30					78	132	56
17	4610	3434	34104			30		3333	333410	3434		200	60				12	272	480	210
18		3434	34410			30		3333	334310	3434		200	60				12	272	480	210
19		3454	3464			30		3333	33346	3454		180	60	12	20			272	480	210
20		3454	3446			30		3333	33436	3454		180	60	12	20			272	480	210
21		3535	3465			30		3333	33356	3535		240		72	20			332	600	270
22		3535	3546			30		3333	33536	3535		240		72	20			332	600	270
23		33444	3434	3454		30		3333	334334	3434	3454	260	120	12				392	660	270
24		3434	34344	3454		30		3333	333344	3434	3454	260	120	12				392	660	270
25		3434	34344	3454		30		3333	333434	3434	3454	260	120	12				392	660	270
26		3434	34344	3454		30		3333	333434	3434	3454	260	120	12				392	660	270
27		33445	3434	3535		30		3333	334335	3434	3535	320	60	72				452	780	330
28		3434	34345	3535		30		3333	333435	3434	3535	320	60	72				452	780	330
29		3434	34435	3535		30		3333	333435	3434	3535	320	60	72				452	780	330
30		3434	34345	3535		30		3333	333345	3434	3535	320	60	72				452	780	330
31	31010	33344	3454		20			333	333344	3445		120	60	12				192	330	140
32		33434	3454		20			333	334334	3454		120	60	12				192	330	140
33		33355	3535		20			333	333355	3535		180		72				252	450	200
34		33535	3535		20			333	335335	3535		180		72				252	450	200
n°	base	corrugated polyhedra			3 ³	3 ⁴	3 ⁵	n1	n2	n3	n4	f3	f4	f5	f6	f8	f10	total		
					complement pyramids			code of composite solids				number of faces by type of polygon						total		

Polyèdres plissés et pyramidés

Les polyèdres plissés et pyramidés sont obtenus par pyramidation des polyèdres plissés de la classe précédente. Dans les cas saturés, à chaque face du solide de support se substitue une pyramide ou une calotte, et le solide sera composé uniquement de polygones de 3, 4 ou 5 côtés. Les caractéristiques de ces polyèdres sont récapitulées au **Tableau 5**.

Corrugated and Stellated Polyhedra

The corrugated and stellated polyhedra are formed by stellating the corrugated polyhedra of the previous section. For saturated polyhedra, where a calotte or a pyramid was glued to each face of the underlying polyhedron, the final solid has only 3, 4 or 5 sided polygons. The characteristics of these polyhedra are summarized in **Table 5**.

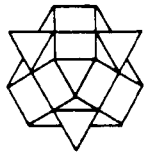


Figure 5.1

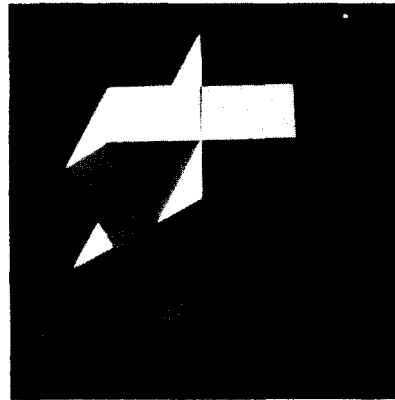


Figure 5.2

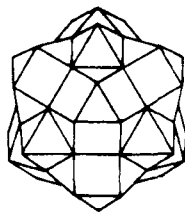
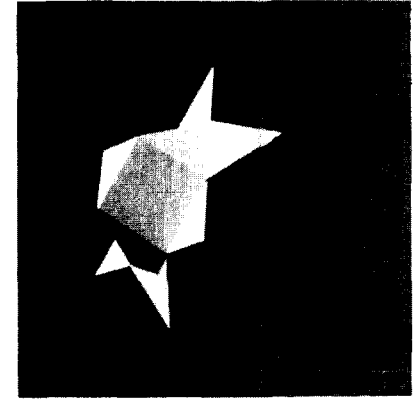


Figure 5.3

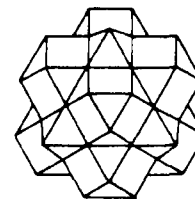
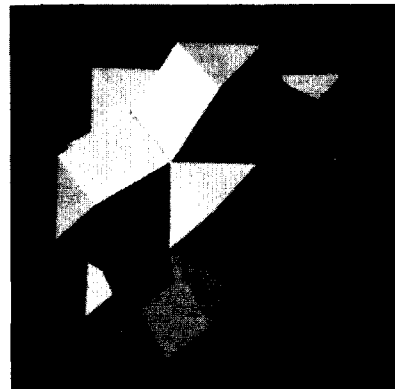
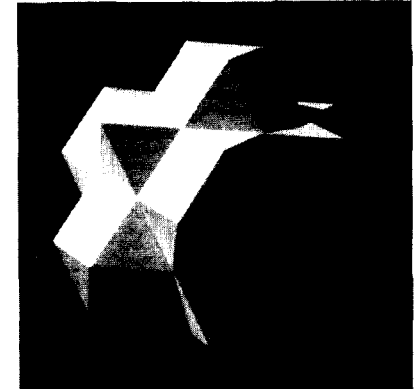


Figure 5.4



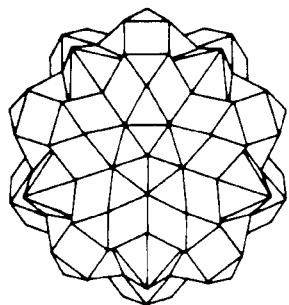


Figure 5.5

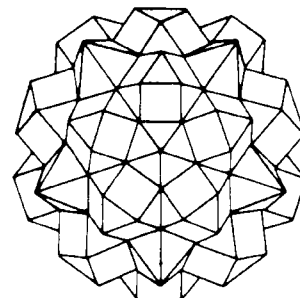
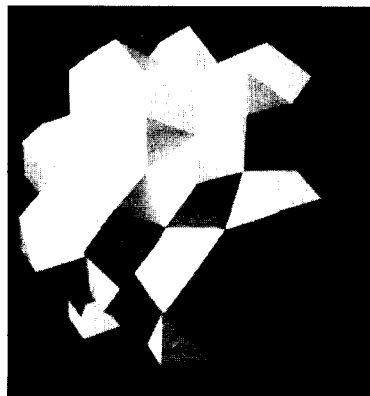


Figure 5.6

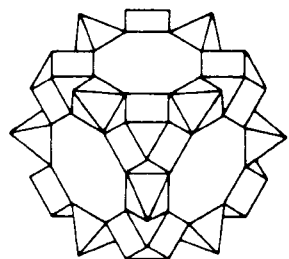
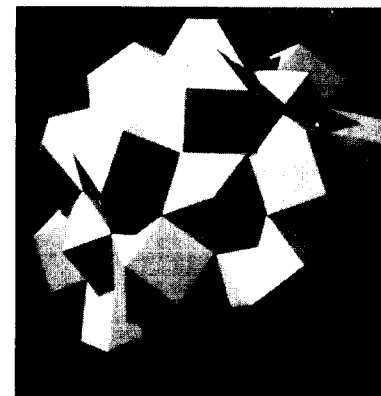


Figure 5.7

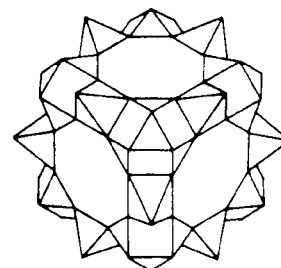
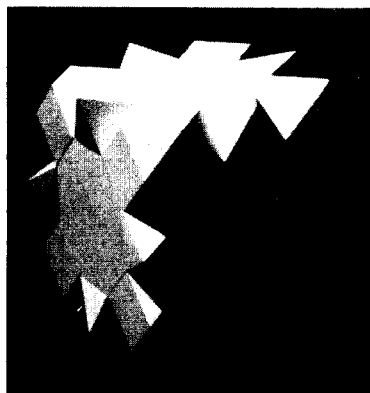


Figure 5.8

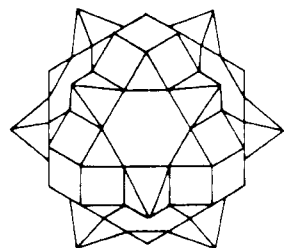
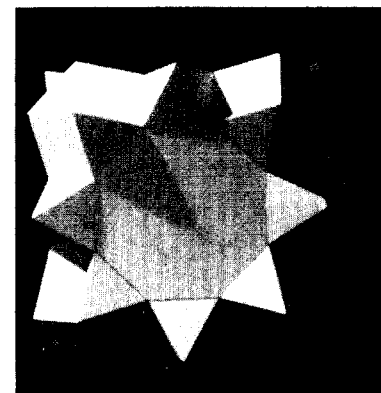


Figure 5.9

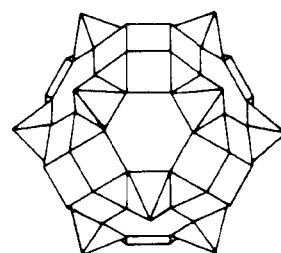
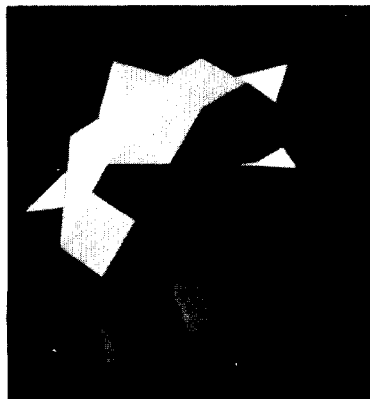
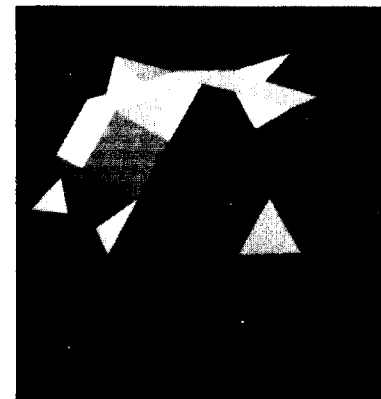


Figure 5.10



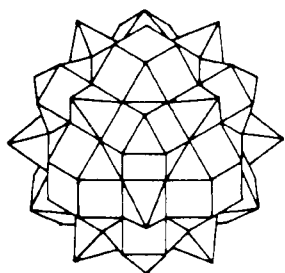


Figure 5.11

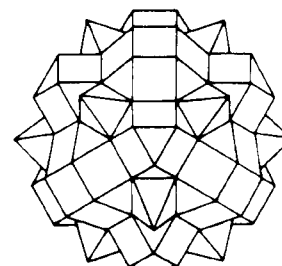
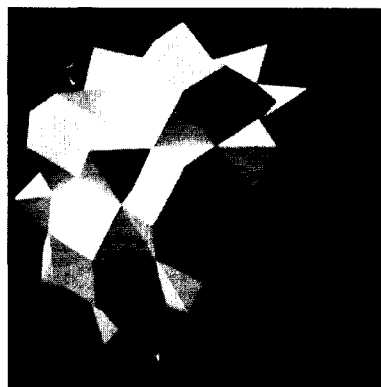


Figure 5.12

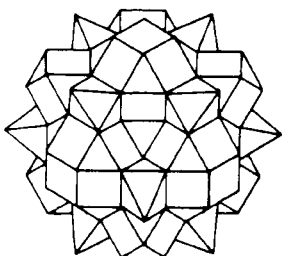
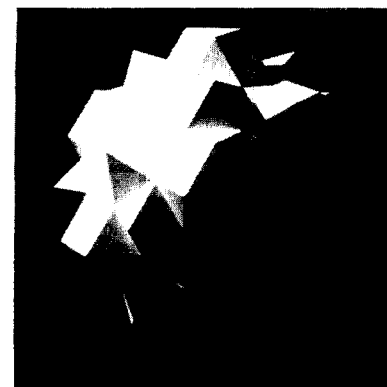


Figure 5.13

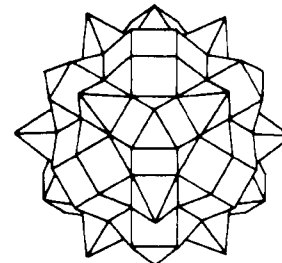
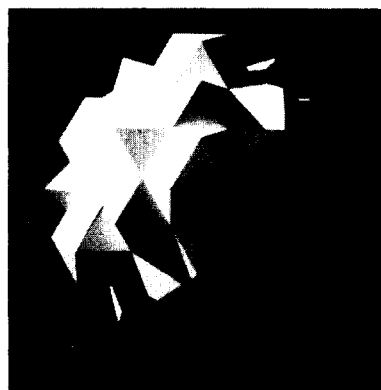


Figure 5.14

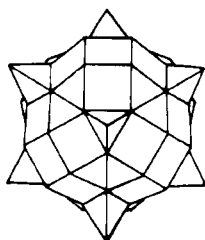
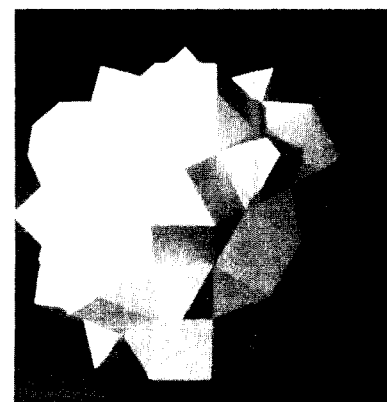


Figure 5.15

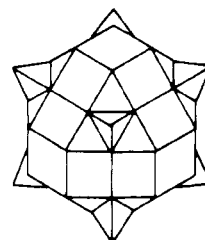
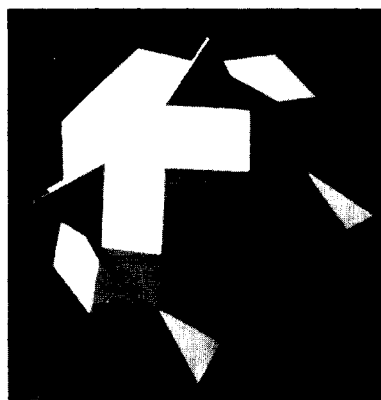
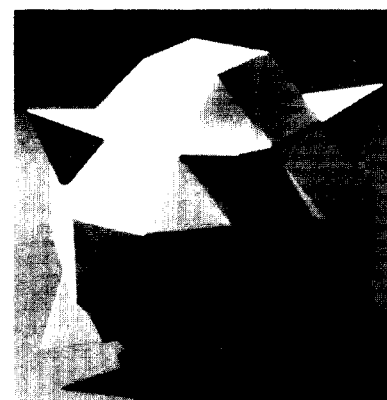


Figure 5.16



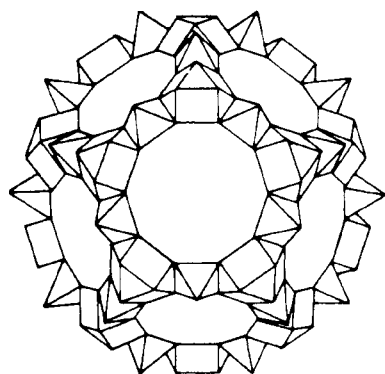


Figure 5.17

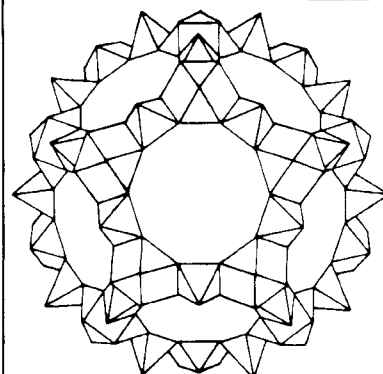
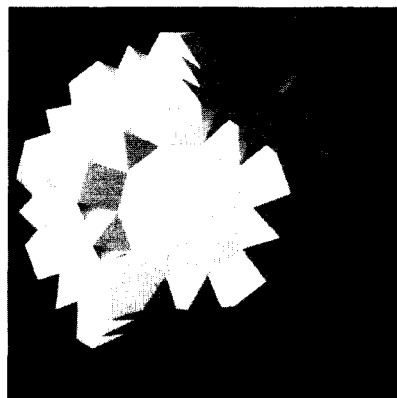


Figure 5.18

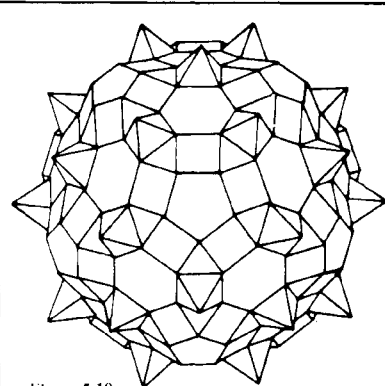
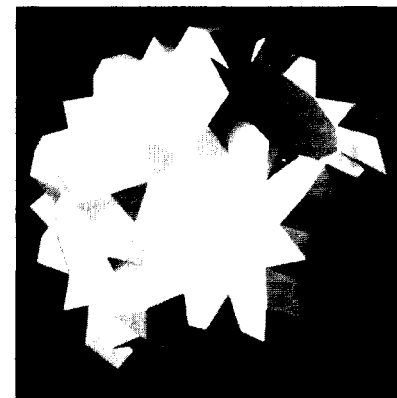


Figure 5.19

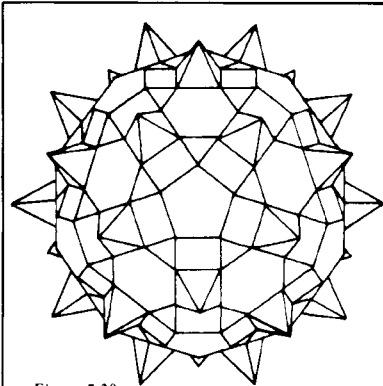
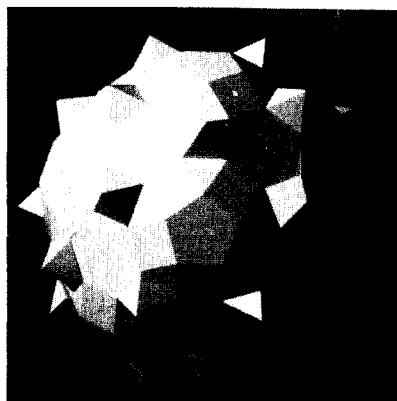


Figure 5.20

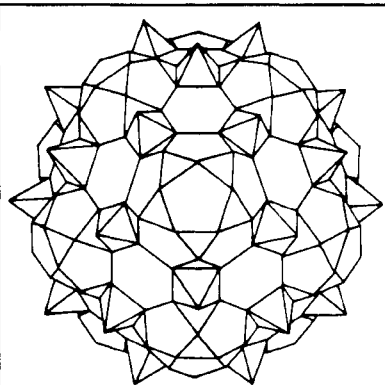
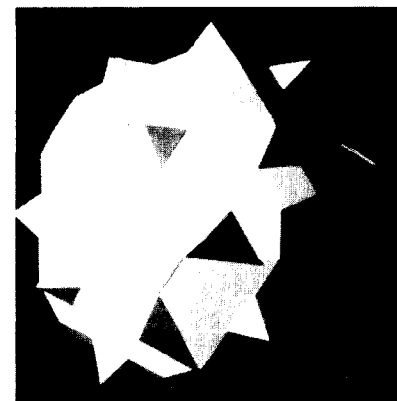


Figure 5.21

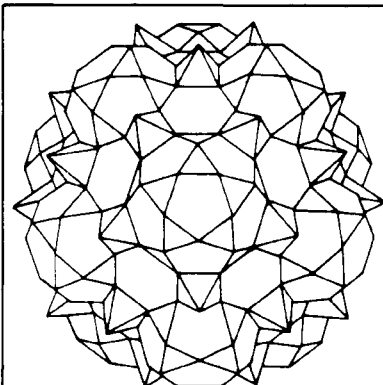
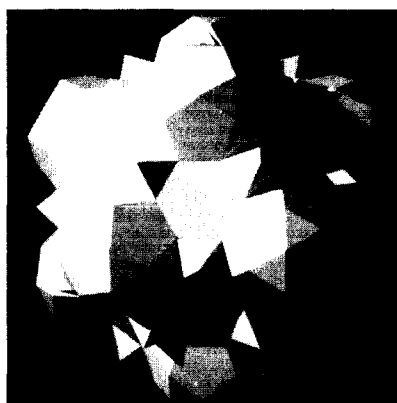
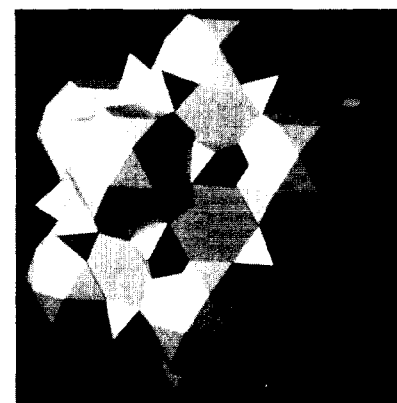


Figure 5.22



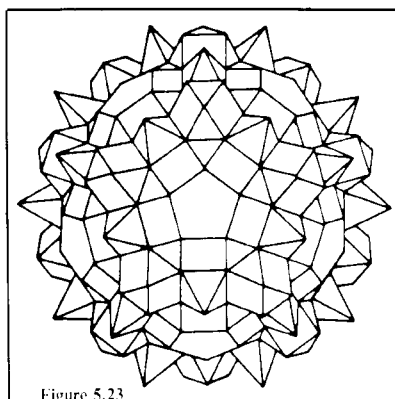


Figure 5.23

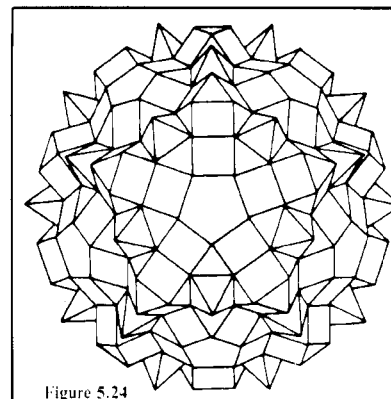
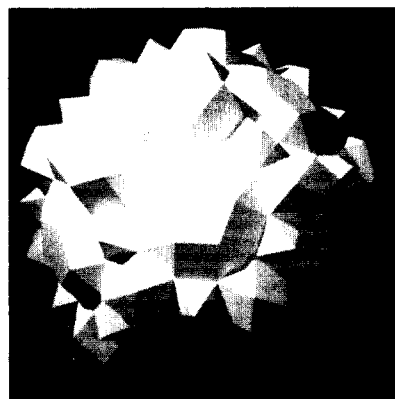


Figure 5.24

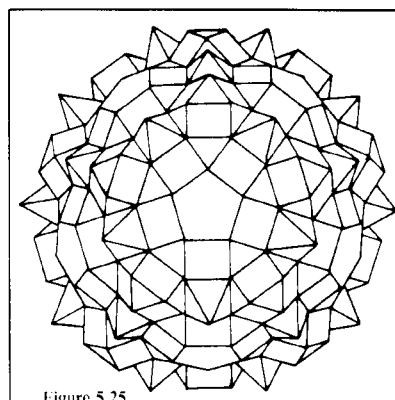
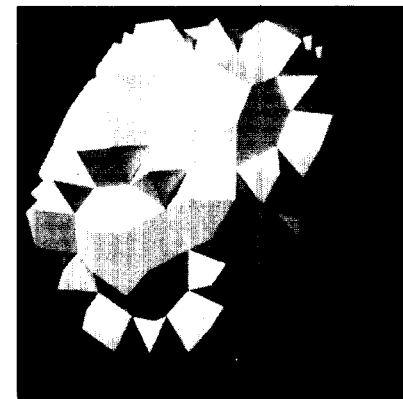


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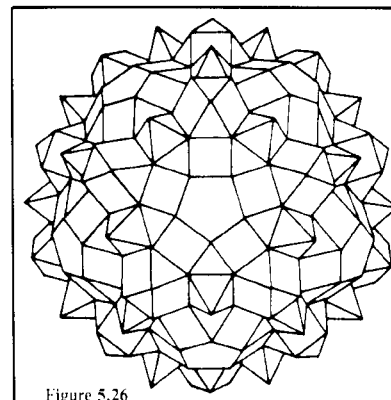
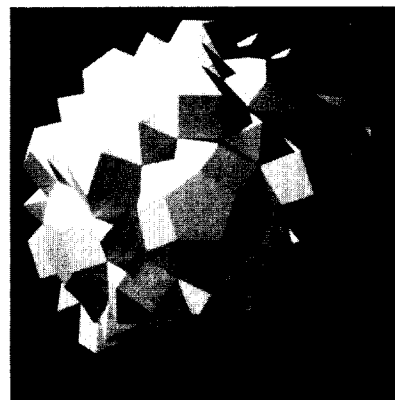


Figure 5.26

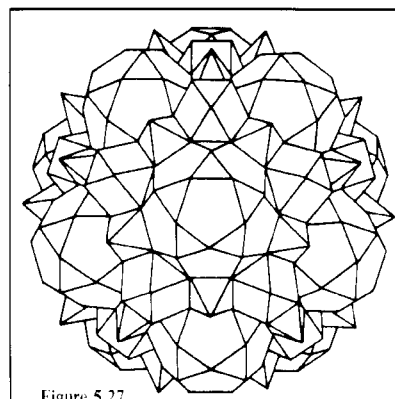
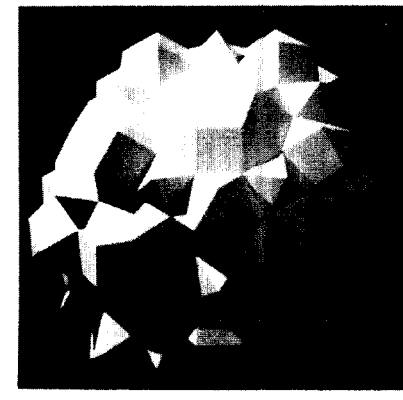


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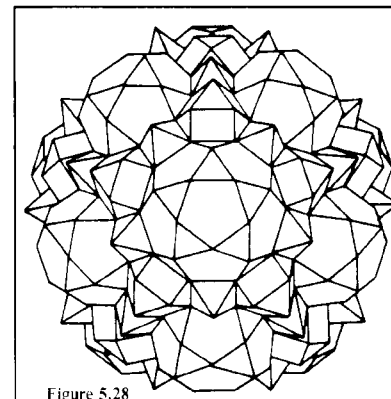
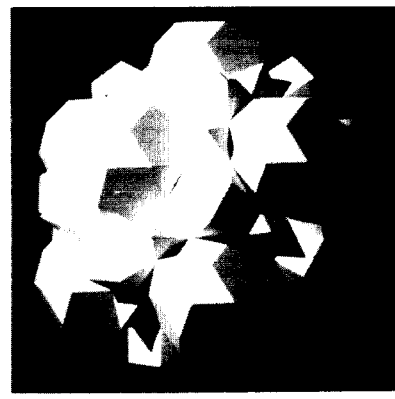
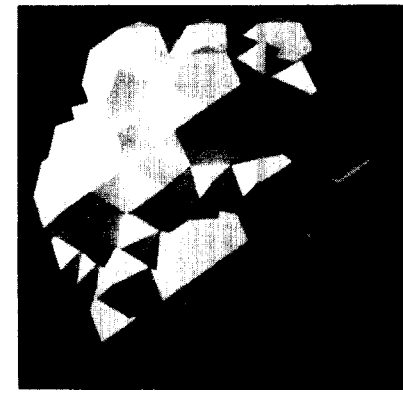


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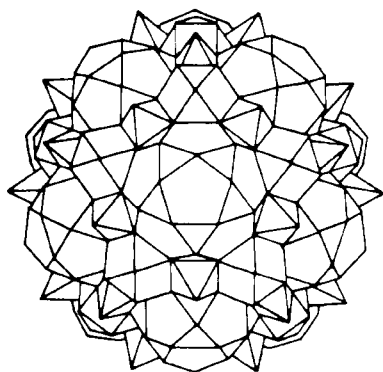


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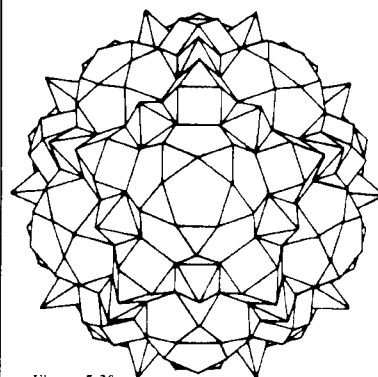
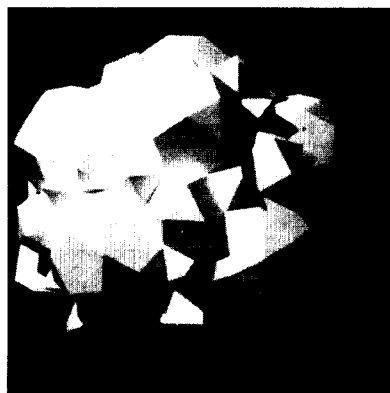


Figure 5.30

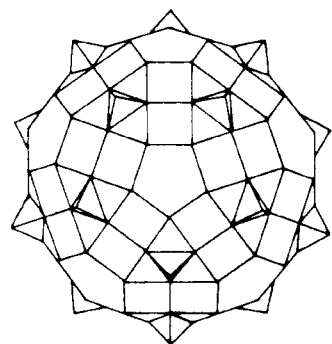


Figure 5.31

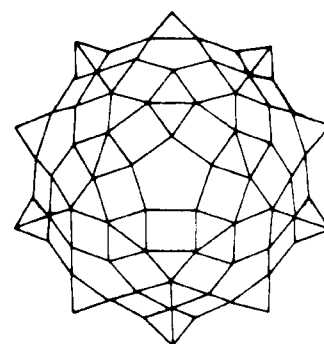
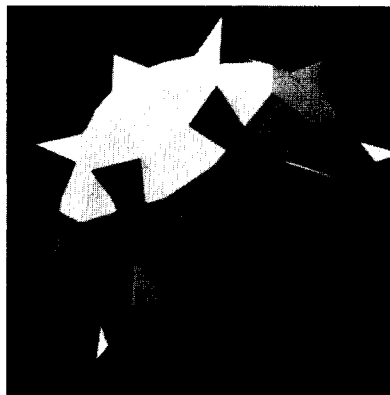


Figure 5.32

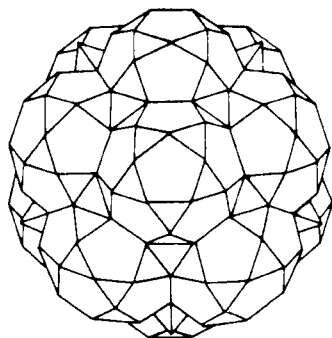
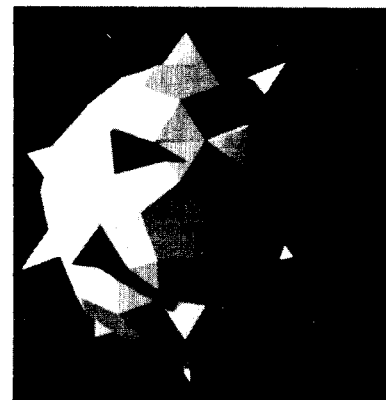


Figure 5.33

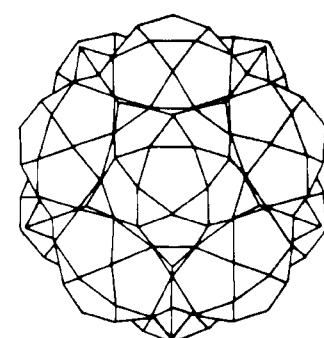
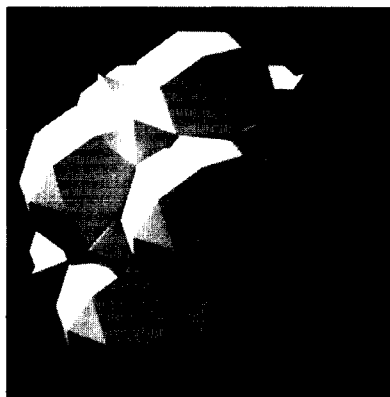
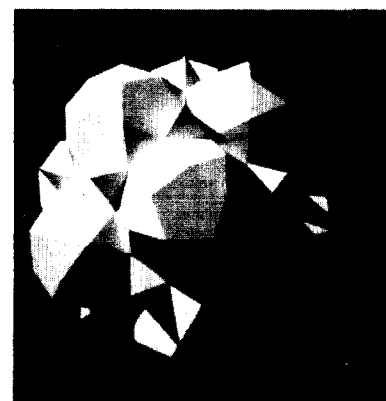


Figure 5.34



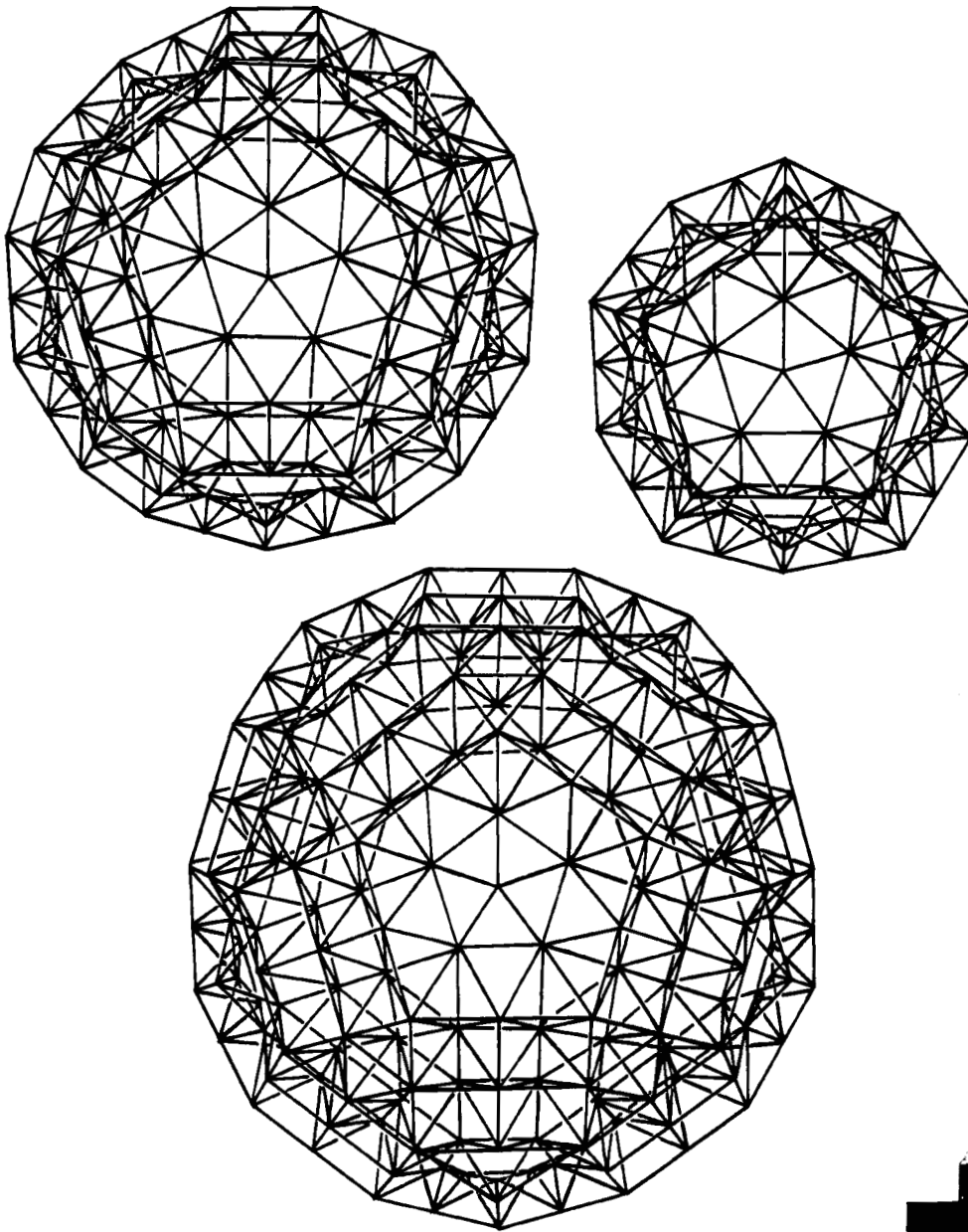


Figure 6 Des coupôles à double nappe • Some double layer cupolas.

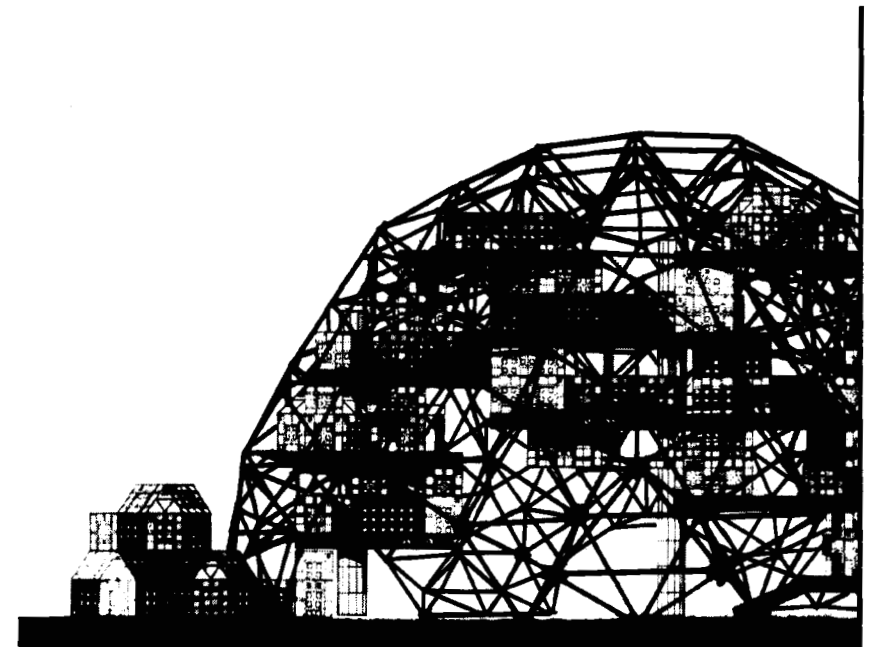


Figure 7 Un bâtiment à remplissage mobile • A building with mobile interior cells.

Des coupoles à double nappe

Partant des polyèdres composites, en reliant leurs sommets extérieurs, on peut obtenir des réseaux convexes à double couche. Dans ces configurations courbes, les arêtes du polyèdre de base forment la couche intérieure et les arêtes des compléments les diagonales intermédiaires, toutes ces membrures étant de longueur identique; tandis que les liaisons ajoutées, qui sont d'une longueur différente, forment la couche extérieure (Emmerich 1965, 1966).

Certains réseaux, notamment ceux dont les faces sont composées en majorité de triangles et de carrés, par exemple le solide pyramidé n° 24, peuvent être distendus pour former des coupoles plus amples où les faces sont des réseaux tétraédriques-octaédriques, appliqués dans les deux positions: triangulaire ou carrée, par conséquent d'une majorité de barres identiques.

Une de ces coupoles à double nappe, d'une grande capacité de portée, a été proposée dans un projet de bâtiment d'habitation à remplissage mobile (concours *CECA* 1965) où le réseau convexe servait de superstructure porteuse de planchers tridimensionnels, appelés *terrains artificiels*, sur lesquels étaient installés des habitacles polyédriques s'agglomérant suivant un empilement rhombicuboctaédrique.

Double Layer Cupolas

Starting with composite polyhedra and joining the exterior vertices we can obtain double layer convex networks. In these curved configurations, the edges of the underlying polyhedron form the inside layer and the edges of the complements form the intermediate diagonals, all these elements being of identical length; on the other hand the added links, which are of different length, form the outside layer (Emmerich 1965, 1966).

Some networks, particularly those with mostly triangular and square faces, for example the pyramidal solid no. 24, can be expanded to form fuller cupolas where the faces are tetrahedral-octahedral trusses, used in both positions: on triangular or square faces, making a majority of the bars the same length.

One of these double layer cupolas with a large span capacity was proposed (competition *CECA* 1965) in a housing project with mobile interior cells where the convex network served as a superstructure carrying threedimensional flooring, called *artificial ground*, in which housing units were installed and joined in a rhombicuboctahedral packing.

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